Morphing Planar Graph Drawings

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Abstract

The study of planar graphs dates back to Euler and the earliest days of graph theory. Centuries later came the proofs by Wagner, Fáry and Stein that every planar graph can be drawn with straight line segments for the edges, and the algorithm by Tutte for constructing such straight-line drawings given in his 1963 paper, "How to Draw a Graph". With more recent attention to complexity issues, this was followed in 1990 by algorithms that construct such drawings on a small grid.

Most people think of "morphing" as a brand new concept, and in fact, the word "morph" was coined in the 80's as a short form of "metamorphose". In common perception, morphing is a high-tech special effect in movies, where, for example, a person's face turns smoothly into a cat's face. We use the term in a more mathematical sense: a *morph* from one drawing of a planar graph to another is a continuous transformation from the first drawing to the second that maintains planarity. Mirroring the developments in planar graphs, the first result was an existence result: between any two planar straight-line graph drawings there exists a morph in which every intermediate drawing is straight-line planar. This was proved surprisingly long ago for triangulations, by Cairns in 1944, and extended to planar graphs by Thomassen in 1983. Both proofs are constructive—they work by repeatedly contracting one vertex to another. Unfortunately, they use an exponential number of steps, and are horrible for visualization purposes since the graph contracts to a triangle and then re-emerges.

The next development was an algorithm to morph between any two planar straight-line drawings, given by Floater and Gotsman in 1999 for triangulations, and extended to planar graphs by Gotsman and Surazhsky in 2001. The morphs are not given by means of explicit vertex trajectories, but rather by means of "snapshots" of the graph at any intermediate time t. By choosing sufficiently many values of t, they give good visual results, but there is no proof that polynomially many steps suffice. Furthermore, the morph suffers from the same drawbacks as Tutte's original planar graph drawing algorithm in that there is no nice bound on the size of the grid needed for the drawings.

For the case of drawing planar graphs the issue of grid size was addressed in 1990 independently by Schnyder and by de Fraysseix, Pach and Pollack, who gave algorithms to construct a straight line planar drawing of any n-vertex planar graph on a grid of size $O(n) \times O(n)$.

The history of morphing planar graph drawings has not progressed to this stage. *It is an open problem to find a polynomial size morph between two given drawings of a planar graph.*

In this survey I will talk about partial and related results. The first main result is that there is a polynomial size morph between any two planar orthogonal graph drawings that preserves planarity and orthogonality (joint work with Mark Petrick and Michael Spriggs). The second main result is that there is a polynomial size morph between any two planar graph drawings if edges are allowed to bend in the intermediate drawings (joint work with Mark Petrick). In both cases the morph consists of a polynomial number of steps, where each step is a *linear morph* that moves vertices along straight-line trajectories.

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