AES BIOFLUX
Advances in Environmental Sciences -
International Journal of the Bioflux Society

Durability of critical infrastructures

Raluca Pascu, Ramiro Sofronie

UNESCO Chair, Faculty of Civil and Environmental Engineering, Bucharest, Romania.
Corresponding author: R. Sofronie, ramiro.sofronie@gmail.com

Abstract. The paper deals with those infrastructures by which world society, under the pressure of demographic explosion, self-survives. The main threatening comes not from terrorist attacks, but from the great natural catastrophes and global climate change. It's not for the first time in history when such measures of self-protection are built up. First objective of this paper is to present the background for durability analysis. Then, with the aid of these mathematical tools the absolute durability of three linear models, typical for critical infrastructures, are successively calculated. In order to enhance the durability of critical infrastructures the solution based on redundancies is chosen. Five types of connection the redundancies for each of the three models are considered. Three topological schemes for connecting the redundancies are adopted: locally, by twining and globally. Absolute values of durability in all fifteen models with redundancies are further calculated. Then, the relative performances of enhanced durability in the same fifteen models, compared with the three original models, considered as references, are analysed. The relative costs of the same fifteen models and in similar topologic conditions are further analysed. By dividing the performance with cost the relative profitableness of each model is obtained. Finally, the three initial models, each reshaped with redundancies in three selective modes, are compared from the perspective of their relative profitableness. The outcomes of this paper are original. They are of practical interests in planning the maintenance programs and checking the plausibility of proposed interventions according to the *clause 7.4* of ISO 13822:2001.
Key Words: cost, performance, profitableness, reliability, serviceability.

Rezumat. Articolul abordează acele infrastructuri prin care societatea mondială, sub presiunea exploziei demografice supravieţuieşte. Principala ameninţare vine nu de la atacurile teroriste, ci de la catastrofele naturale şi schimbările climatice globale. Nu este prima dată în istorie când sunt luate astfel de măsuri de protecţie. Primul obiectiv al lucrării este prezentarea cadrului de analiză a durabilităţii. Apoi, cu ajutorul acestor instrumente matematice, sunt calculate succesiv durabilitatea absolută a trei modele liniare, tipice pentru infrastructurile critice. Pentru a spori durabilitatea infrastructurilor critice, este aleasă soluţia pe baza redundanţelor. Se iau în considerare cinci tipuri de conexiuni de redundanţe pentru fiecare dintre cele trei modele. Sunt adoptate trei scheme topologice pentru conectarea redundanţelor: locale, prin twining şi globale. Valorile absolute ale durabilităţii în toate cele cincisprezece modele cu redundanţe sunt calculate mai departe. Apoi sunt analizate performanţele relative ale durabilităţii crescute în aceleaşi cincisprezece modele, comparate cu cele trei modele originale considerate ca referinţe. Sunt analizate costurile relative ale aceloraşi cincisprezece modele şi în condiţii topologice similare. Prin împărţirea performanţei la cost, se obţine profitabilitatea relativă a fiecărui model. În final, cele trei modele iniţiale, fiecare reconturat cu redundanţele în trei moduri selective, sunt comparate din perspectiva profitabilităţii lor relative. Rezultatele acestei lucrări sunt originale şi sunt de un interes practic în susţinerea clauzei 7.4 privind verificarea plauzibilităţii intervenţiilor de mentenanţă propuse, conform *clauzei 7.4* din ISO 13822:2001.
Cuvinte cheie: cost, performanţă, profitabilitate, fiabilitate, serviabilitate.

1. Introduction. According to statistics of the United Nations during the last five decades only, from 1960 to 2000, the world population increased three times, reaching in 2010 the total amount of 6,868,000,000 people (Fig. 1). The "Day of 7 Billion" has been targeted for July 2012. Naturally, on the world map more spots that are crowded gradually appear (Fig. 2). If in 1800, only 3% of the world's population lived in cities, at the end of the 20th Century the rate of urbanization increased to 47%. During this demographic explosion, a large number of assets have been created. They are essential for the functioning of a society and economy during that dramatic explosion. The list of main assets contains electricity generation, transmission and distribution, gas production, transport and distribution, oil and oil products production, transport and distribution, telecommunication, water supply, transportation systems, food production and distribution, heating systems, public health, financial services and security services as well (Hamaker 1982; Dordea & Coman 2007; Petrescu-Mag 2009). Claiming the peril of terrorist attacks, politicians called the above-mentioned assets as *critical infrastructures* and *key resources*. Since they have a surviving role for society, they were also classified as *vital assets*. Consequently, governments are competing in issuing national and regional strategies for providing the most sophisticated systems of top security. The cost of all these efforts is increasing at least with the same rate as the number of population is increasing.

While on the ground world population is irrepressibly rising, around the Earth different physical phenomena occur. Generally, they have a dynamic character and are governed by the universal laws of equilibrium. When by chance they come in contact with human settlements or activities suddenly these phenomena are producing disasters and therefore were called natural hazards. Theory of safety defines the hazard as the probability that a site or region is prone to losses and fatalities. The Organization of United Nations is deeply concerned with the great natural catastrophes, namely in the events regionally extended when the affected countries become depended for surviving and recovery by international assistance. Following the ancient philosophy of the four basic Elements to live in harmony: *Earth, Air*, *Water,* and *Fire*, the United Nations classified the natural hazards in *geophysical, meteorological*, *hydrological*, and *climatologic events*. 1) Geophysical events refer to earthquakes and volcanic eruptions; 2) Meteorological events consider tropical storms, winter storms, severe weather, hail, tornados, and local storms; 3) Hydrological events include river flood, flash flood, storm surge, and mass movements as landslides; 4) Climatologic events refer to heat wave, freeze, wild land fire and drought. Three of the four groups also include the consequences of climate change (Pascu 2009).

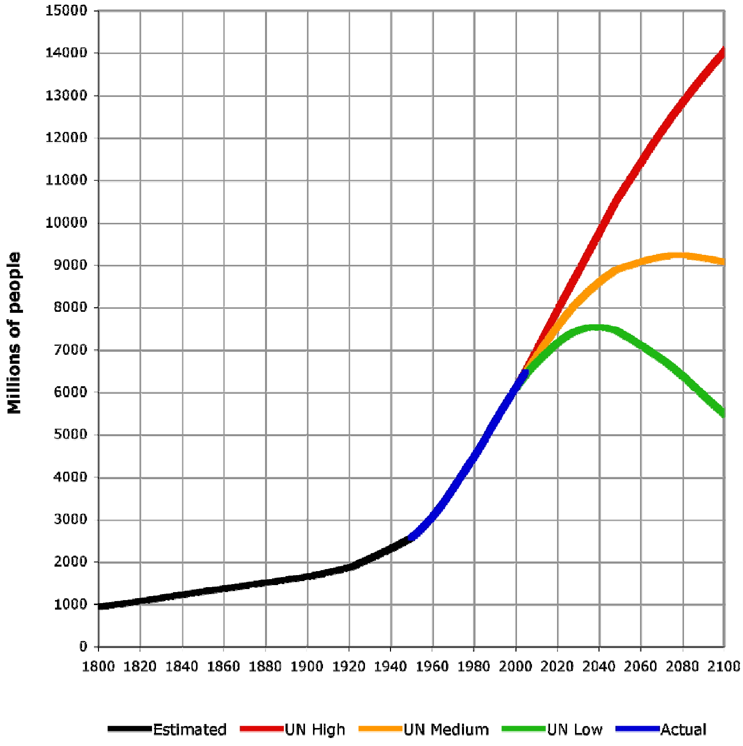

Figure 1. World population growth from 1800 to 2100 (http://en.wikipedia.org/wiki/)

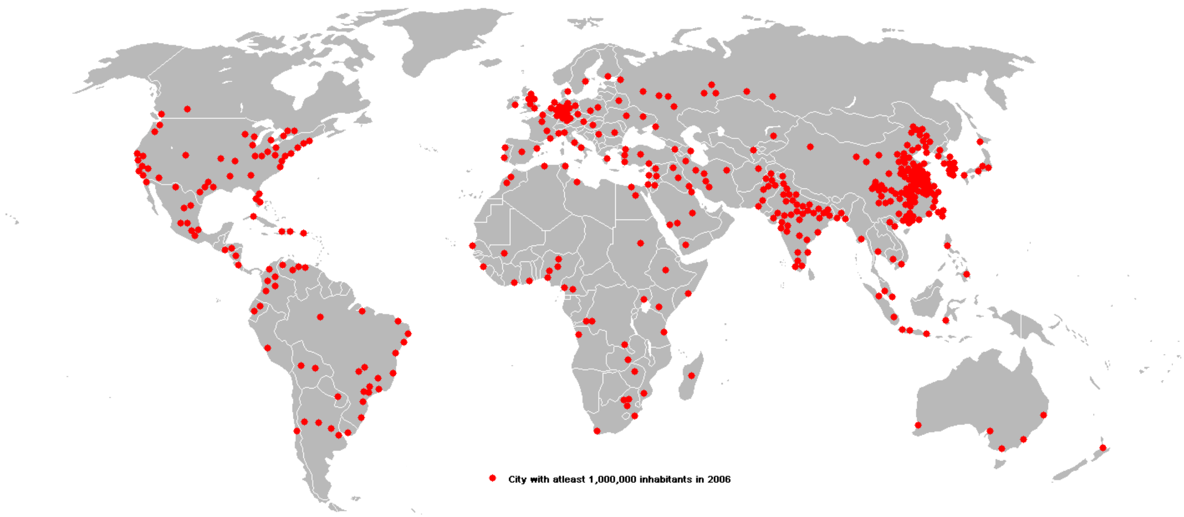

Figure 2. Mega-cities in 2006 (http://en.wikipedia.org/wiki/World_population)

Recent statistics of the great natural catastrophes, that occurred during the 1950-2008 period, covering a period of almost 60 years. They mention 285 major events, which means an average of 4.75 disasters per year, 2,000,000 fatalities namely 34,482 death per year, and the value of total losses reaching 1.97 billion US$ or 34 million US$ per year (Munich Re 2010). In addition, from the total number 285 of major events the most of them, in proportion of 41%, were of meteorological nature, earthquakes in proportion of 53% caused the most fatalities, and the values of losses were due to both storms in proportion of 38% and earthquakes in proportion of 33% (Fig. 3). It is to be noticed that according to statistics the number of major events has an increasing trend, while their effects are extending in space and superposing in time according to the saying *a major event never comes alone* (Sofronie 2010) (Fig. 4).

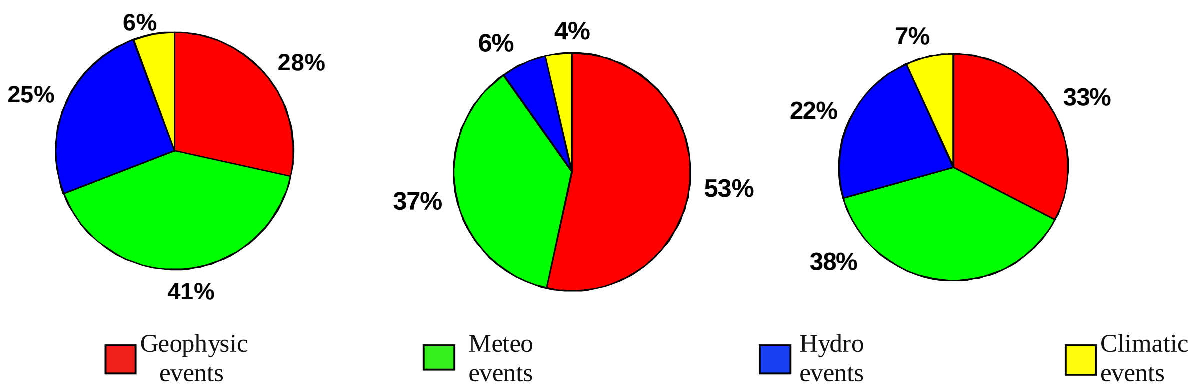

Figure 3. Number of events, fatalities and losses due to natural catastrophes during 1950-2008 (Munich Re 2010).

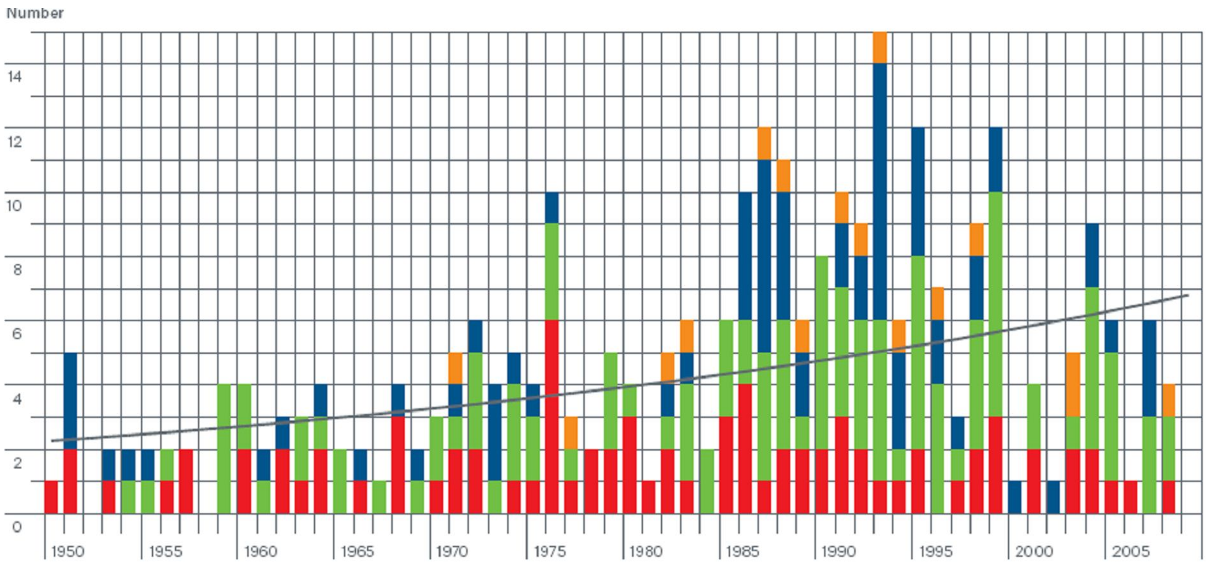

Figure 4. The number of great natural catastrophes in the world in period 1950-2008 (Munich Re 2010).

Even if the critical infrastructures were created by the modern society for surviving, the invention of such structures with similar function of self-protection is as old as human history. All ancient cities, for instance, were provided with defense walls. The Great Wall of China was built in 5th Century BC as a long fortress. It stretches for 10,000 Li or 8,851.8 km and was used for this purpose as far as the 16th century AD (Fig. 5). On Romanian territory there are also some vestiges lasting since Roman Empire or from time that is more recent. For instance, the Church of Probota Monastery in Bukovina, built in 1530 by Prince Petru Rares, the son of Stephan the Great, was provided with a defense wall measuring 4x90=360 m in length and 6.0 m in height. Due to its outstanding paintings well preserved up today and to its ecclesiastic architecture, since 1993 the Church was inscribed on UNESCO World Heritage List under the number 397 (Fig. 6).

Figure 5. The Great Wall of China (http://en.wikipedia.org/wiki/Great_Wall_of_China/)

Figure 6. Defense wall of Probota Monastery
(http://www.romanianmonasteries.org/)

All the assets listed above as critical infrastructures are in fact constructions as creations of Civil Engineering. Transportation systems of people, goods and materials, energy and water, for instance, are linear constructions while health, financial and security services are networks of buildings. According to *Eurocode 1, Part 1, Basis of Design, clauses 2.1(1)P* and *2.1(3)P* all constructions, named in legislation as *structures,* together with their *structural members,* should fulfill three fundamental requirements: 1) Serviceability limit state; 2) Ultimate limit state; and 3) Robustness, i.e. the ability of not being damaged by events such as fire or explosions or consequences of human errors. The same *Eurocode 1* by *clause 2.5(1)* defines the *durability* of a structure as its ability to remain fit for use during the design working life given appropriate maintenance. In addition, according to *clause 2.4*, performance criteria are the principal requirement to be considered in the overall strategy for achieving durability (Gulvanessian 1996).

The first objective of this paper is to present the background for analysing the durability of critical infrastructures (ISO 2394, 1988). Then, with the aid of these mathematical tools, the absolute durability of three linear models, each with three, four or five elements, typical for critical infrastructures, are successively calculated (ISO 13822, 2010). Unfortunately, due to connections in series, the obtained values are far to fulfill the fundamental requirements. In order to enhance the durability of critical infrastructures an effective and flexible solution seems to be that based on redundancies. Five types of connection the redundancies for each of the three models are chosen. Therefore, to the original three models without redundancies other fifteen models with different redundancies are analysed. Three topological schemes for connecting the redundancies are chosen: 1) locally, to a single element, by adding one, two or three redundancies; 2) twining all elements, one by one, with redundancies; 3) globally, attaching one redundancy in parallel over all elements of each of the three models. For sake of simplicity, all redundancies assume the same capacity like the other elements of models. The absolute values of durability in all fifteen models with redundancies are further calculated. Then, the relative performances of enhancing the durability in the same fifteen models compared with the three original models, considered as references, are analysed. The relative costs of the same models and in similar topological conditions are further analysed. Since the two parameters regarding performance and cost follow discordant evolutions, the third parameter is necessary. It is the relative profitableness obtained by dividing the performance by cost. Finally, the three original models with three, four and five models, each reshaped with redundancies in three selective modes, are compared from the perspective of their relative profitableness.

2. Analysis background. Mathematical Theory of Reliability defines *the durability* by *mid time between failures*, abbreviated by MTBF. Either *hour, day* or *year* are used as units for MTBF which is expressed by the integral equation

$$\tau = \int_0^\infty F(t)dt \quad , (1)$$

where in the most general case the reliability function for a single element has the form

$$F(t) = e^{-\int_0^t \lambda(t)dt} \quad , (2)$$

and where $\lambda(t)$ is the risk function measured either in *1/hour, 1/day* or *1/year.* Graphically, equation (2) assumes the so-called *tub shape* (Fig. 7)

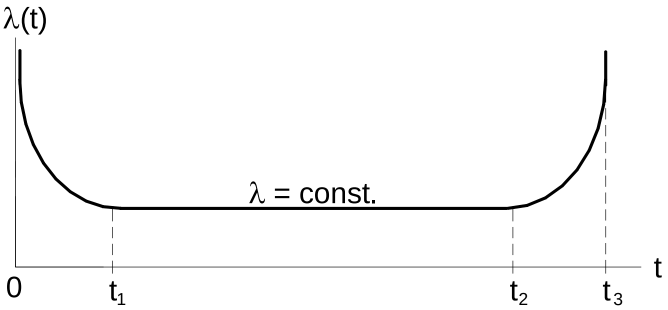

Figure 7. Tub-shaped diagram of risk function.

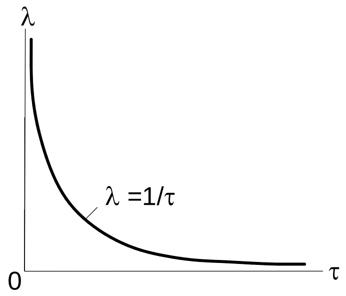

Figure 8. Durability-risk relation.

Particularly, in the case of a constant risk

$$\lambda = const. \quad , (3)$$

and when the two laws of the chance, due to Georges-Louis Leclerc, Comte de Buffon (1707-1788), subsist, reliability equation (2) simplifies and becomes

$$F(t) = e^{-\lambda t} \quad , (4)$$

and sometimes could assume Taylor's expansion in series

$$F(t) = 1 - \lambda t + \frac{(\lambda t)^2}{2!} - \frac{(\lambda t)^3}{3!} + ... \quad , (5)$$

Between durability and risk factor the relation of inverse proportionality

$$\tau = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda} \quad , (6)$$

represented by an equilateral hyperbola, is established (Fig. 8)
In the case of a set of elements connected in series and which obeys the first law of the chance, namely: *something and something and something*... the function of reliability used in equation (1) takes one of the following forms

$$F_s(t) = F_1(t)F_2(t)...F_n(t) = \prod_{i=1}^{n} F_i(t) = e^{-\sum_{i=1}^{n}\int_0^t \lambda_i(t)dt} \cong \prod_{i=1}^{n}[1 - \int_0^t \lambda_i(t)dt] \quad , (7)$$

For instance, the reliability function of the set composed by two identical elements connected in series assumes the expression

$$F_s(t) = F_1F_2 = F_0^2 = e^{-\lambda t} \quad , (8)$$

and its durability will be

$$\tau_s = \int_0^\infty F_s(t)dt = \frac{1}{2\lambda} = 0.5\tau_o$$, (9)

where F_0 and τ_0 are the individual reliability and durability of each element, respectively.

In the case of a set of elements connected in parallel and which obeys the second law of the chance, namely: *something or something or something...* the function of reliability used in expression (1) takes one of the following forms

$$F_p(t) = 1 - D_1(t)D_2(t)...D_n(t) = 1 - \prod_{i=1}^{n}[1 - F_i(t)] = 1 - \prod_{i=1}^{n}[1 - e^{-\int_0^t \lambda_i(t)dt}] \cong 1 - \prod_{i=1}^{n}\lambda_i t^n$$, (10)

in which $D_i(t)$ is the complementary function of reliability defined by the fundamental equation

$$F(t) + D(t) = 1$$, (11)

For instance, the reliability function of a set of two identical elements composed by two identical elements connected in parallel assumes the expression

$$D_p = D_1D_2 \Rightarrow (1 - F_p) = (1 - F_1)(1 - F_2) = (1 - F_0)^2 \Rightarrow F_p = 2F_0 - F_0^2 = 2e^{-\lambda t} - e^{-2\lambda t}$$ (12)

and its durability will be

$$\tau_p = \int_0^\infty F_p(t)dt = \frac{2}{\lambda} - \frac{1}{2\lambda} = \frac{3}{2\lambda} = 1.5\tau_o$$, (13)

three times longer than that of the former set in series.

For sets with both kinds of connection, in series and parallel called mixed sets, appropriate combinations between the equations (7) and (10) are used. For instance, if the human body is regarded as a random set with identical elements having the individual durability $\tau_0 = 12 hours$, two alternative models can be successively analysed:

1). The model of physical work consisting of one element representing the body (B) connected in series with two elements in parallel representing the arms (A) (Fig. 9)

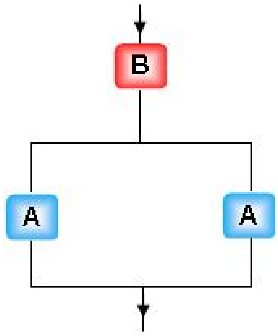

Figure 9. Model of the physical work.

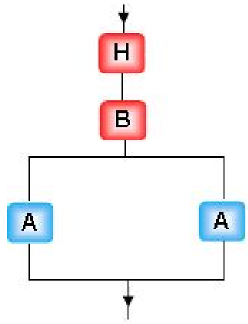

Figure 10. Model of the intellectual work.

The reliability of physical model assumes the expression

$$F_{phy}(t) = F_0F_p = F_0(2F_0 - F_0^2) = 2F_0^2 - F_0^3$$, (14)

and its durability becomes

$$\tau_{phy} = \int_0^\infty F_{phy}(t)dt = \int_0^\infty (2e^{-2\lambda t} - e^{-3\lambda t})dt = \frac{2}{2\lambda} - \frac{1}{3\lambda} = \frac{4}{6\lambda} = \frac{2x12}{3} = 8 hours$$, (15)

namely the very value provided by the International Labour Organisation in Geneva.

2). The model of intellectual work consisting of two elements in series, one representing the head (H) and another the body (B), connected with two elements in parallel representing the arms (A) (Fig. 10)

The reliability of intellectual model assumes the expression

$$F_{\text{int}}(t) = F_0^2 F_p = F_0^2(2F_0 - F_0^2) = 2F_0^3 - F_0^4 \quad , (16)$$

and its durability becomes

$$\tau_{\text{int}} = \int_0^\infty F_{\text{int}}(t)dt = \int_0^\infty (2e^{-3\lambda t} - e^{-4\lambda t})dt = \frac{2}{3\lambda} - \frac{1}{4\lambda} = \frac{5}{12\lambda} = \frac{5x12}{12} = 5hours \quad , (17)$$

Further, the reliability analysis can be similarly developed at any level of accuracy or for any particular details. The aim of this paper is to extend the durability of critical infrastructures with the aid of redundancies as much as their serviceability limit allows.

3. Series of three elements. 3.1 Analysis model of the reference series of three identical elements, each with the individual risk λ=1/1000 day, is presented below (Fig. 11)

Figure 11. Reference series of three elements.

The reliability of this series assumes the expression

$$F_1(t) = F_o^3 = e^{-3\lambda t} \quad , (18)$$

and its durability takes in this case the value

$$\tau_1 = \int_0^\infty F_1(t)dt = \int_0^\infty e^{-3\lambda t}dt = \frac{1}{3\lambda} = \frac{1000}{3} = 333days < \tau_o = 1{,}000days \quad , (19)$$

3.2 Analysis model of the series of three elements with a single redundancy of the same value like the others and locally connected in parallel to one element is presented below (Fig. 12)

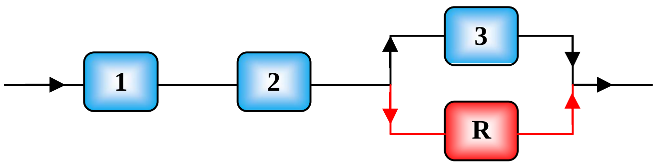

Figure 12. Redundancy locally connected in parallel with a series of three elements.

The reliability of this series assumes the expression

$$F_2(t) = F_0^2 F_p(R=1) \quad , (20)$$

where

$$1 - F_p = (1 - F_0)^2 = 1 - 2F_0 + F_0^2$$

and with

$$F_p(R=1) = 2F_0 - F_0^2 \quad , (21)$$

one finds

$$F_2(t) = F_0^2(2F_0 - F_0^2) = 2F_0^3 - F_0^4 = 2e^{-3\lambda t} - e^{-4\lambda t} \quad . (22)$$

Durability of the series takes in this case the value

$$\tau_2 = \int_0^\infty F_2(t)dt = \int_0^\infty (2e^{-3\lambda t} - e^{-4\lambda t})dt = \frac{2}{3\lambda} - \frac{1}{4\lambda} = \frac{5}{12\lambda} = \frac{5x1000}{12} = 417days \quad , (23)$$

3.3 Analysis model of the series of three elements with two redundancies of the same value like the others and locally connected in parallel to one element is presented below (Fig. 13)

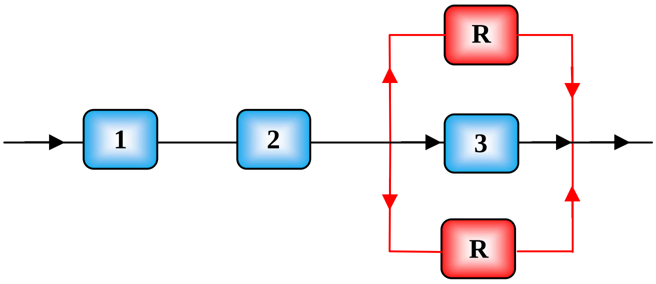

Figure 13. Two redundancies locally connected in parallel with a series of three elements.

The reliability of this series assumes the expression

$$F_3(t) = F_0^2 F_p(R=2) \quad , (24)$$

where

$$1 - F_p = (1 - F_0)^3 = 1 - 3F_0 + 3F_0^2 - F_0^3$$

and with

$$F_p(R=2) = 3F_0 - 3F_0^2 + F_0^3 \quad , (25)$$

one finds

$$F_3(t) = F_0^2(3F_0 - 3F_0^2 + F_0^3) = 3F_0^3 - 3F_0^4 + F_0^5 = 3e^{-3\lambda t} - 3e^{-4\lambda t} + e^{-5\lambda t} \quad , (26)$$

Durability of the series takes in this case the value

$$\tau_3 = \int_0^\infty F_3(t)dt = \int_0^\infty (3e^{-3\lambda t} - 3e^{-4\lambda t} + e^{-5\lambda t})dt = \frac{3}{3\lambda} - \frac{3}{4\lambda} + \frac{1}{5\lambda} = \frac{27}{60\lambda} = \frac{27x1000}{60} = 450 days \quad , (27)$$

3.4 Analysis model of the series of three elements with three redundancies of the same value like the others and locally connected in parallel to one element is presented below (Fig. 14)

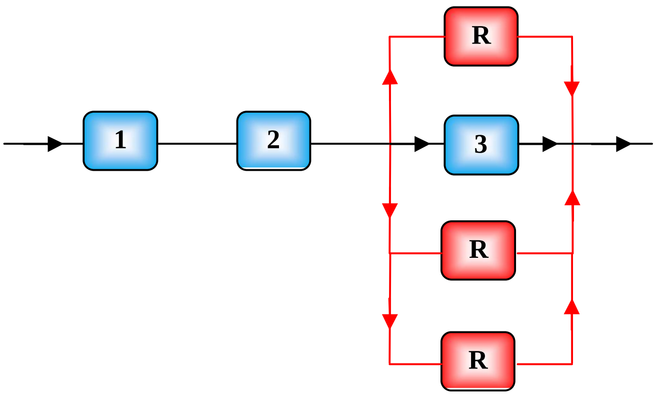

Figure 14. Three redundancies locally connected in parallel with a series of three elements.

The reliability of this series assumes the expression

$$F_4(t) = F_0^2 F_p(R=3) \quad , (28)$$

where

$$1 - F_p = (1 - F_0)^4 = 1 - 4F_0 + 6F_0^2 - 4F_0^3 + F_0^4$$

and

$$F_p(R=3) = 4F_0 - 6F_0^2 + 4F_0^3 - F_0^4 \text{ , (29)}$$

one finds

$$F_4(t) = F_0^2(4F_0 - 6F_0^2 + 4F_0^3 - F_0^4) = 4F_0^3 - 6F_0^4 + 4F_0^5 - F_0^6 = 4e^{-3\lambda t} - 6e^{-4\lambda t} + 4e^{-5\lambda t} - e^{-6\lambda t} \text{ , (30)}$$

Durability of the series in this case takes the value

$$\tau_4 = \int_0^\infty F_4(t)dt = \int_0^\infty (4e^{-3\lambda t} - 6e^{-4\lambda t} + 4e^{-5\lambda t} - e^{-6\lambda t})dt = \frac{4}{3\lambda} - \frac{6}{4\lambda} + \frac{4}{5\lambda} - \frac{1}{6\lambda} = \frac{28}{60\lambda} = 467 days \text{ , (31)}$$

3.5 Analysis model of the series of three elements with three redundancies of the same value like the others and connected in parallel one by one with each element is presented below (Fig. 15)

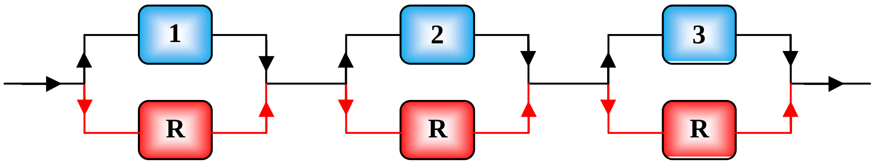

Figure 15. Three redundancies connected in parallel by twining series elements.

The reliability of this series assumes the expression

$$F_5(t) = F_p^3(R=1) = (2F_0 - F_0^2)^3 = 8F_0^3 - 12F_0^4 + 6F_0^5 - F_0^6 = 8e^{-3\lambda t} - 12e^{-4\lambda t} + 6e^{-5\lambda t} - e^{-6\lambda t} \text{ ,(32)}$$

and durability of the series takes in this case the value

$$\tau_5 = \int_0^\infty F_5(t)dt = \int_0^\infty (8e^{-3\lambda t} - 12e^{-4\lambda t} + 6e^{-5\lambda t} - e^{-6\lambda t})dt = \frac{8}{3\lambda} - \frac{12}{4\lambda} + \frac{6}{5\lambda} - \frac{1}{6\lambda} = \frac{42}{60\lambda} = 700 days \text{ , (33)}$$

3.6 Analysis model of the series of three elements with a single redundancy of the same value like the other three and globally connected in parallel to the series is presented below (Fig. 16)

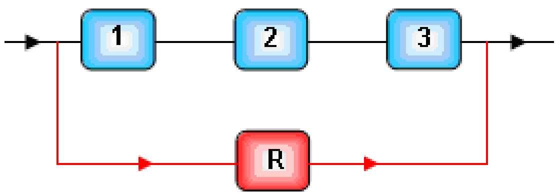

Figure 16. Redundancy globally connected in parallel with a series of three elements.

The reliability of this series assumes the expression

$$F_6(t) = F_0 + F_0^3 - F_0^4 = e^{-\lambda t} + e^{-3\lambda t} - e^{-4\lambda t} \text{ , (34)}$$

and durability of the series takes in this case the value

$$\tau_6 = \int_0^\infty F_6(t)dt = \int_0^\infty (e^{-\lambda t} + e^{-3\lambda t} - e^{-4\lambda t})dt = \frac{1}{\lambda} + \frac{1}{3\lambda} - \frac{1}{4\lambda} = \frac{13}{12\lambda} = \frac{13x1000}{12} = 1083 days > \tau_o \text{ , (35)}$$

It is for the first time in this case 3.1 when the value of individual durability τ_0 was over-passed by the durability of the whole model composed of three elements in series.
Further, the relative performance of the model with three elements in series and different redundancies is analysed with the ratio

$$P_i = \left(\frac{\tau_i}{\tau_1}\right)_{i=1}^{i=6}, \quad (36)$$

where τ_1 is the reference durability, then the relative cost of the model defined by ratio

$$C_i = \left(\frac{n_i}{n_1}\right)_{i=1}^{i=6}, \quad (37)$$

where the reference number of elements is in this case $n_1 = 3$, and finally, the index of profitableness

$$\eta_i = \left(\frac{P_i}{C_1}\right)_{i=1}^{i=6}, \quad (38)$$

divides the performance value with the cost of models. The results are summarised below in a comparative table (Table 1).

Table 1

Series of three elements with redundancies

Model number	*Number elements*	*Number of redundancies*	*Absolute durability*	*Relative performance*	*Relative cost*	*Relative profitableness*
1	3	0	333 days	1,00	1,00	1,00
2	4	1	417 days	1,25	1,33	0,94
3	5	2	450 days	1,35	1,67	0,81
4	6	3	467 days	1,40	2,00	0,70
5	6	3	700 days	2,10	2,00	1,05
6	4	1	1083 days	3,25	1,33	2,44

The diagram of absolute durability of the series with three elements shows increasing values from a model to another suggesting that the effects of redundancies are positive (Fig. 17)

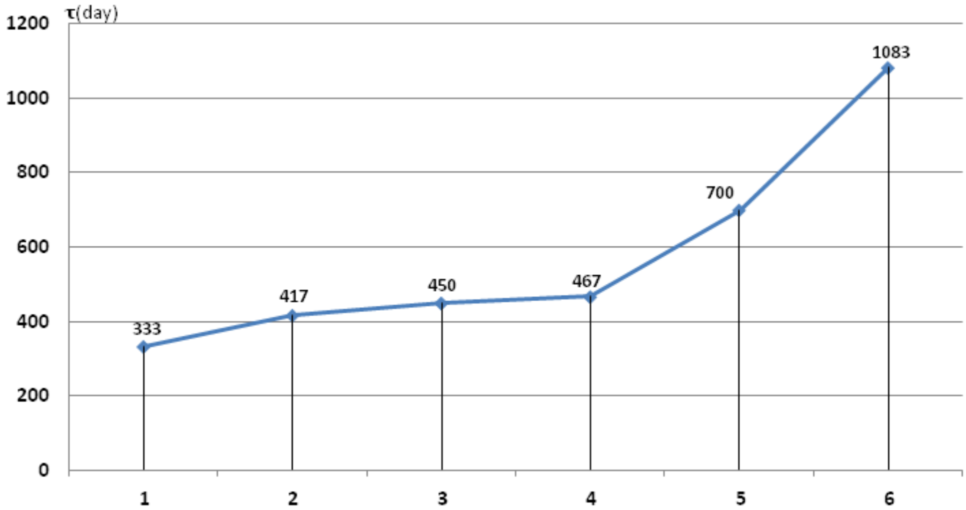

Figure 17. Absolute durability of the series with three elements in the six analysis models.

The diagram of relative profitableness shows that in the first three case the values are decreasing and only in the subsequent two cases increasing values are reached, the last one being substantial (Fig. 18).

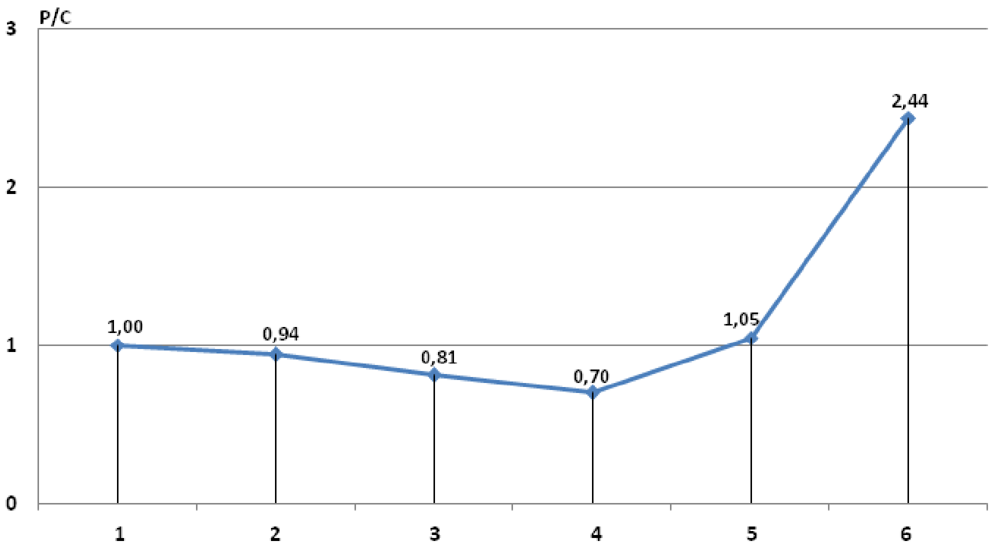

Figure 18. Relative profitableness for the series with three elements in the six analysis models.

4. Series of four elements

4.1 Analysis model of the reference series of four identical elements, each with the individual risk λ=1/1000 day, is presented below (Fig. 19).

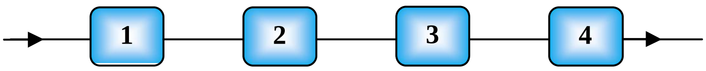

Figure 19. Reference series of four elements.

The reliability of this series assumes the expression

$$F_1(t) = F_o^4 = e^{-4\lambda t} \quad , (39)$$

and its durability takes in this case the value

$$\tau_1 = \int_0^\infty F_1(t)dt = \int_0^\infty e^{-4\lambda t}dt = \frac{1}{4\lambda} = \frac{1000}{4} = 250days \quad , (40)$$

4.2 Analysis model of the series of four elements with a single redundancy of the same value like the others and locally connected in parallel to one element is presented below (Fig. 20)

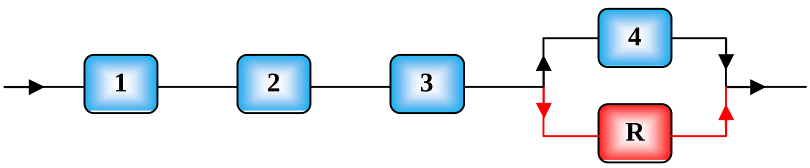

Figure 20. Redundancy locally connected in parallel with a series of four elements.

The reliability of this series assumes the expression

$$F_2(t) = F_0^3 F_p(R=1) = F_0^3(2F_0 - F_0^2) = 2F_0^4 - F_0^5 = 2e^{-4\lambda t} - e^{-5\lambda t} \quad , (41)$$

and its durability takes in this case the value

$$\tau_2 = \int_0^\infty F_2(t)dt = \int_0^\infty (2e^{-4\lambda t} - e^{-5\lambda t})dt = \frac{2}{4\lambda} - \frac{1}{5\lambda} = \frac{3}{10\lambda} = \frac{3x1000}{10} = 300days \quad , (42)$$

4.3 Analysis model of the series of four elements with two redundancies of the same value like the others and locally connected in parallel to one element is presented below (Fig. 21)

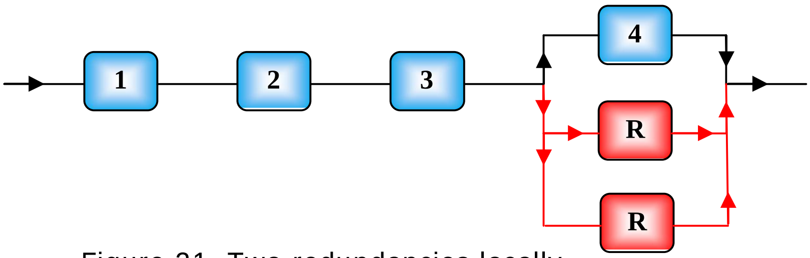

Figure 21. Two redundancies locally connected in parallel with a series of four elements.

The reliability of this series assumes the expression

$$F_3(t) = F_0^3 F_p(R=2) = F_0^3(3F_0 - 3F_0^2 + F_0^3 = 3F_0^4 - 3F_0^5 + F_0^6 = 3e^{-4\lambda t} - 3e^{-5\lambda t} + e^{-6\lambda t} \quad , (43)$$

and its durability takes in this case the value

$$\tau_3 = \int_0^\infty F_3(t)dt = \int_0^\infty (3e^{-4\lambda t} - 3e^{-5\lambda t} + e^{-6\lambda t})dt = \frac{3}{4\lambda} - \frac{3}{5\lambda} + \frac{1}{6\lambda} = \frac{19}{60\lambda} = \frac{19x1000}{60} = 317 days \quad , (44)$$

4.4 Analysis model of the series of four elements with three redundancies of the same value like the others and locally connected in parallel to one element is presented below (Fig. 22).

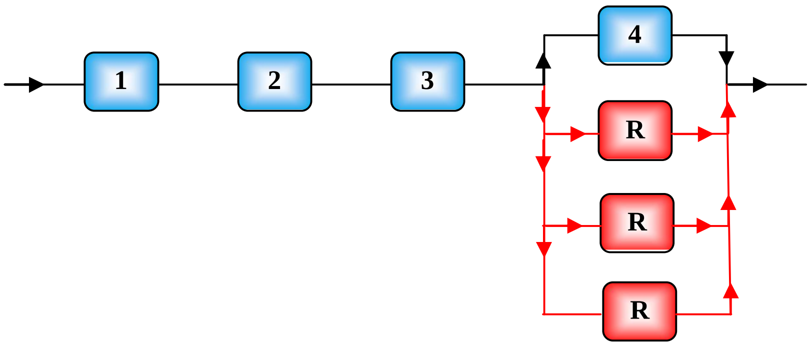

Figure 22. Three redundancies locally connected in parallel with a series of four elements.

The reliability of this series assumes the expression

$$F_4(t) = F_0^3 F_p(R=3) = F_0^3(4F_0 - 6F_0^2 + 4F_0^3 - F_0^4) = 4F_0^4 - 6F_0^5 + 4F_0^6 - F_0^7 \quad , (45)$$

and its durability takes in this case the value

$$\tau_4 = \int_0^\infty F_4(t)dt = \int_0^\infty (4e^{-4\lambda t} - 6e^{-5\lambda t} + 4e^{-6\lambda t} - e^{-7\lambda t})dt = \frac{4}{4\lambda} - \frac{6}{5\lambda} + \frac{4}{6\lambda} - \frac{1}{7\lambda} = \frac{68x1000}{210} = 323 days$$

(46)

4.5 Analysis model of the series of four elements with four redundancies of the same value like the others and connected in parallel one by one with each element is presented below (Fig. 23).

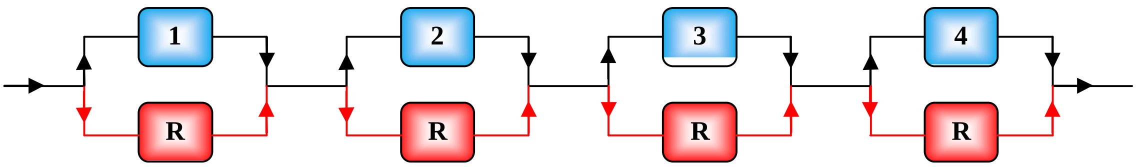

Figure 23. Four redundancies connected in parallel by twining series elements.

The reliability of this series assumes the expression

$$F_5(t) = F_p^4(R=1) = (2F_0 - F_0^2)^4 = 16F_0^4 - 32F_0^5 + 24F_0^6 - 8F_0^7 + F_0^8 \quad , (47)$$

and its durability takes in this case the value

$$\tau_5 = \int_0^\infty F_5(t)dt = \int_0^\infty (16e^{-4\lambda t} - 32e^{-5\lambda t} + 24e^{-6\lambda t} - 8e^{-7\lambda t} + e^{-8\lambda t})dt = \frac{16}{4\lambda} - \frac{32}{5\lambda} + \frac{24}{6\lambda} - \frac{8}{7\lambda} + \frac{1}{8\lambda} = \frac{163}{280\lambda} =$$

$$\frac{163x1000}{280} = 582 days \quad , (48)$$

4.6 Analysis model of the series of four elements with a single redundancy of the same value like the other four and globally connected in parallel to the series is presented below (Fig. 24)

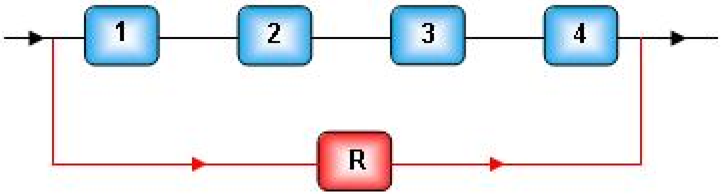

Figure 24. Redundancy globally connected in parallel with a series of four elements

The reliability of this series assumes the expression

$$F_6(t) = F_0 + F_0^4 - F_0^5 = e^{-\lambda t} + e^{-4\lambda t} - e^{-5\lambda t} \quad , (49)$$

and its durability takes in this case the value

$$\tau_6 = \int_0^\infty F_6(t)dt = \int_0^\infty (e^{-\lambda t} + e^{-4\lambda t} - e^{-5\lambda t})dt = \frac{1}{\lambda} + \frac{1}{4\lambda} - \frac{1}{5\lambda} = \frac{21}{20\lambda} = \frac{21x1000}{20} = 1050 zile \quad , (50)$$

Like in the previous chapter, further relative performances of the series with four elements and different redundancies were analysed. Then, the relative cost of each model with redundancies was calculated. Finally, profitableness index, by dividing each performance with its corresponding cost, was found. The results are summarised below in a comparative table (Table 2).

Table 2

Series of four elements with redundancies

Model number	*Number elements*	*Number of redundancies*	*Absolute durability*	*Relative performance*	*Relative cost*	*Relative profitableness*
1	4	0	250 days	1,00	1,00	1,00
2	5	1	300 days	1,20	1,25	0,96
3	6	2	317 days	1,27	1,50	0,85
4	7	3	323 days	1,29	1,75	0,74
5	8	4	582 days	2,33	2,00	1,17
6	5	1	1050 days	4,20	1,25	3,37

The diagram of absolute durability of the series with four elements shows increasing values from a model to another suggesting that the effects of redundancies are positive (Fig. 25).

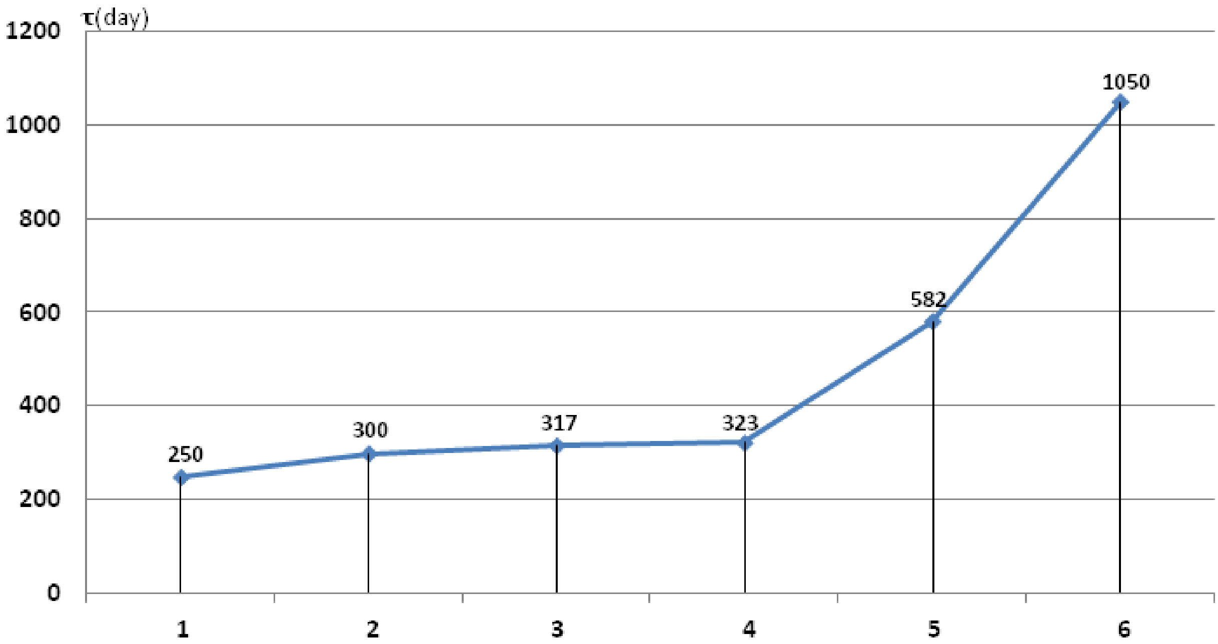

Figure 25. Absolute durability of the series with four elements in the six analysis models.

The diagram of relative profitableness shows that in the first three cases the values are decreasing and only in the subsequent two cases increasing values are reached, the last one being substantial (Fig. 26).

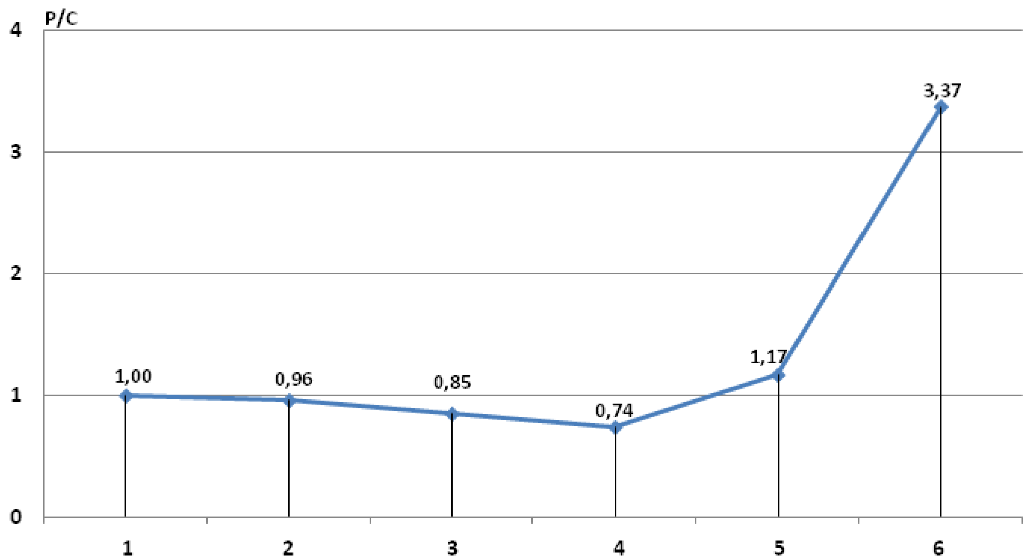

Figure 26. Relative profitableness for the series with four elements in the six analysis models.

5. Series of five elements

5.1. Analysis model of the reference series of five identical elements, each with the individual risk $\lambda=1/1000$ day, is presented below (Fig. 27).

Figure 27. Reference series of five elements.

The reliability of this series assumes the expression

$$F_1(t) = F_o^5 = e^{-5\lambda t}$$, (51)

and its durability takes in this case the value

$$\tau_1 = \int_0^\infty F_1(t)dt = \int_0^\infty e^{-5\lambda t}dt = \frac{1}{5\lambda} = \frac{1000}{5} = 200 days$$, (52)

5.2. Analysis model of the series of five elements with a single redundancy of the same value like the others and locally connected in parallel to one element is presented below (Fig. 28).

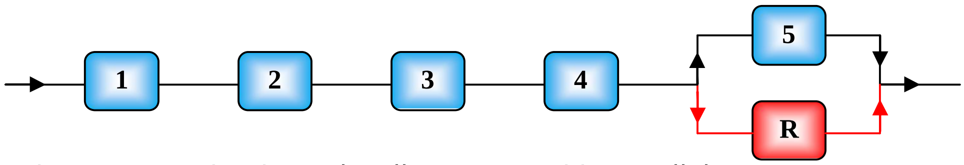

Figure 28. Redundancy locally connected in parallel with a series of five elements.

The reliability of this series assumes the expression

$$F_2(t) = F_0^4 F_p(R=1) = F_0^4(2F_0 - F_0^2) = 2F_0^5 - F_0^6$$, (53)

and its durability takes in this case the value

$$\tau_2 = \int_0^\infty F_2(t)dt = \int_0^\infty (2e^{-5\lambda t} - e^{-6\lambda t})dt = \frac{2}{5\lambda} - \frac{1}{6\lambda} = \frac{7}{30\lambda} = \frac{7x1000}{30} = 233 days$$. (54)

5.3. Analysis model of the series of five elements with two redundancies of the same value like the others and locally connected in parallel to one element is presented below (Fig. 29).

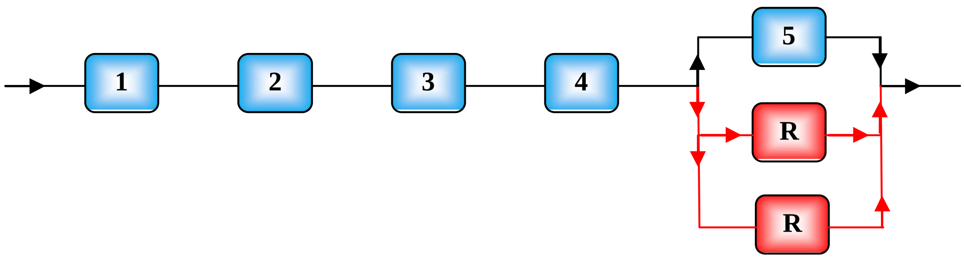

Figure 29. Two redundancies locally connected in parallel with a series of five elements.

The reliability of this series assumes the expression

$$F_3(t) = F_0^4 F_p(R=2) = F_0^4(3F_0 - 3F_0^2 + F_0^3) = 3F_0^5 - 3F_0^6 + F_0^7$$, (55)

and its durability takes in this case the value

$$\tau_3 = \int_0^\infty F_3(t)dt = \int_0^\infty (3e^{-5\lambda t} - 3e^{-6\lambda t} + e^{-7\lambda t})dt = \frac{3}{5\lambda} - \frac{3}{6\lambda} + \frac{1}{7\lambda} = \frac{51}{210\lambda} = \frac{51x1000}{210} = 243 days$$,(56)

5.4 Analysis model of the series of four elements with three redundancies of the same value like the others and locally connected in parallel to one element is presented below (Fig. 30).

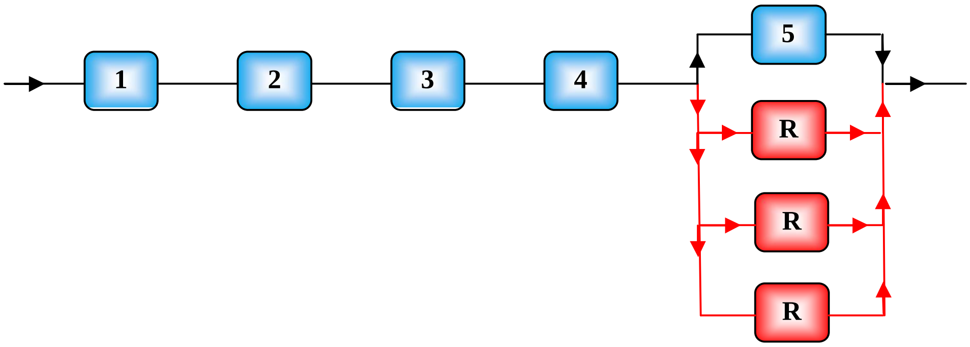

Figure 30. Three redundancies locally connected in parallel with a series of five elements.

The reliability of this series assumes the expression

$$F_4(t) = F_0^4 F_p(R=3) = F_0^4(4F_0 - 6F_0^2 + 4F_0^3 - F_0^4) = 4F_0^5 - 6F_0^6 + 4F_0^7 - F_0^8 \quad , \text{(57)}$$

and its durability takes in this case the value

$$\tau_4 = \int_0^\infty F_4(t)dt = \int_0^\infty (4e^{-5\lambda t} - 6e^{-6\lambda t} + 4e^{-7\lambda t} - e^{-8\lambda t})dt = \frac{4}{5\lambda} - \frac{6}{6\lambda} + \frac{4}{7\lambda} - \frac{1}{8\lambda} = \frac{69}{280\lambda} = 246 days \quad , \text{(58)}$$

5.5 Analysis model of the series of five elements with five redundancies of the same value like the others and connected in parallel one by one with each element is presented below (Fig. 31)

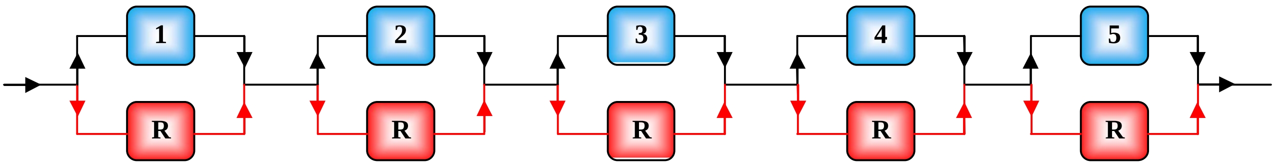

Figure 31. Five redundancies connected in parallel by twining series elements.

The reliability of this series assumes the expression

$$F_5(t) = F_p^5(R=1) = (2F_0 - F_0^{2)}) = 32F_0^5 - 80F_0^6 + 80F_0^7 - 40F_0^8 + 10F_0^9 - F_0^{10} \quad , \text{(59)}$$

and its durability takes in this case the value

$$\tau_5 = \int_0^\infty F_5(t)dt = \int_0^\infty (32e^{-5\lambda t} - 80e^{-6\lambda t} + 80e^{-7\lambda t} - 40e^{-8\lambda t} + 10e^{-9\lambda t} - e^{-10\lambda t})dt = \frac{9570x1000}{18900} = 506 days \quad \text{(60)}$$

5.6 Analysis model of the series of four elements with a single redundancy of the same value like the other four and globally connected in parallel to the series is presented below (Fig. 32)

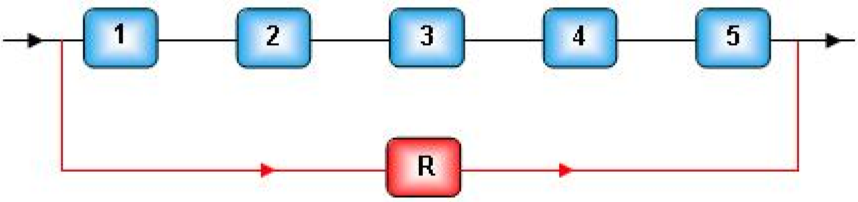

Figure 32. Redundancy globally connected in parallel with a series of five elements.

The reliability of this series assumes the expression

$$F_6(t) = F_0 + F_0^5 - F_0^6 = e^{-\lambda t} + e^{-5\lambda t} - e^{-6\lambda t} \quad , (61)$$

and its durability takes in this case the value

$$\tau_6 = \int_0^\infty F_6(t)dt = \int_0^\infty (e^{-\lambda t} + e^{-5\lambda t} - e^{-6\lambda t})dt = \frac{1}{\lambda} + \frac{1}{5\lambda} - \frac{1}{6\lambda} = \frac{31}{30\lambda} = \frac{31x1000}{30} = 1033 days \quad , (62)$$

Like in the previous chapter further relative performances of the series with four elements and different redundancies were analyzed. Then, the relative cost of each model with redundancies was calculated. Finally, profitableness index, by dividing each performance with its corresponding cost, is presented. The results are summarised below in a comparative table (Table 3).

Table 3

Series of five elements with redundancy

Model number	Number elements	Number of redundancies	Absolute durability	Relative performance	*Relative cost*	*Relative profitableness.*
1	5	0	200 days	1,00	1,00	1,00
2	6	1	233 days	1,17	1.20	0,98
3	7	2	243 days	1,22	1,40	0,87
4	8	3	246 days	1,23	1,60	0,77
5	10	5	506 days	2,53	2,00	1,27
6	6	1	1033 days	5,17	1,20	4,31

The diagram of absolute durability of the series with five elements shows increasing values from a model to another suggesting that the effects of redundancies are positive (Fig. 33).

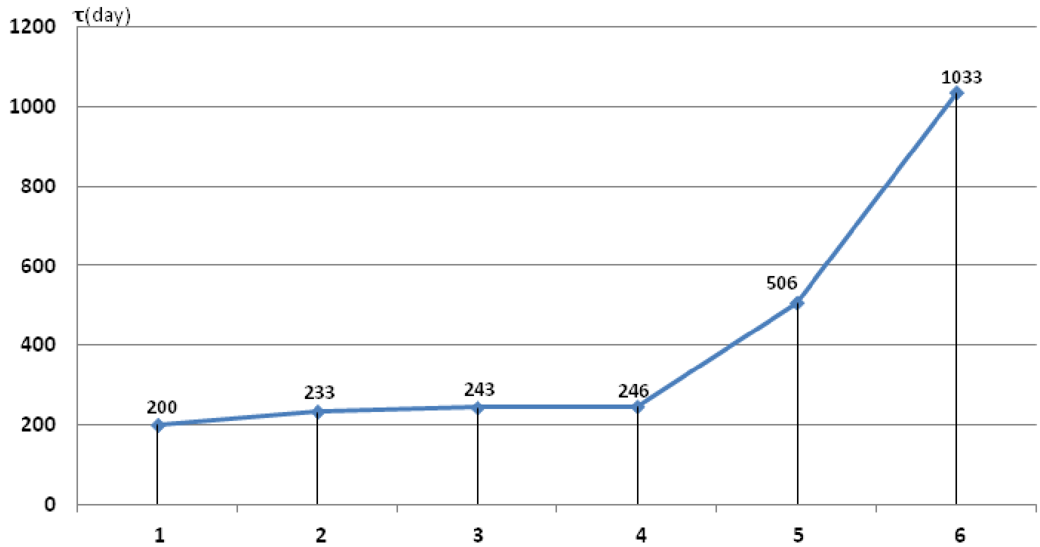

Figure 33. Absolute durability of the series with five elements in the six analysis models.

The diagram of relative profitableness shows that in the first three cases the values are decreasing and only in the subsequent two cases increasing values are reached, the last one being substantial (Fig. 34).

6. Analysis outputs. The efficiency of redundancies in enhancing the durability of critical infrastructures is best convincing by the profitableness displayed by each topological model. In the former chapters, three types of series with three, four and five elements have been successively analysed. For each series, five topological schemes of redundancy connections are considered. Of those five schemes, only three proved to be relevant for profitableness analysis, namely: 1) local parallel connection to a single element of series; 2) connection in parallel to each element of series by twining; and 3) global connection in parallel of one redundancy to all elements of series. The results of comparative analysis are suggestive and therefore for an easily understanding are graphically presented below (Fig. 35).

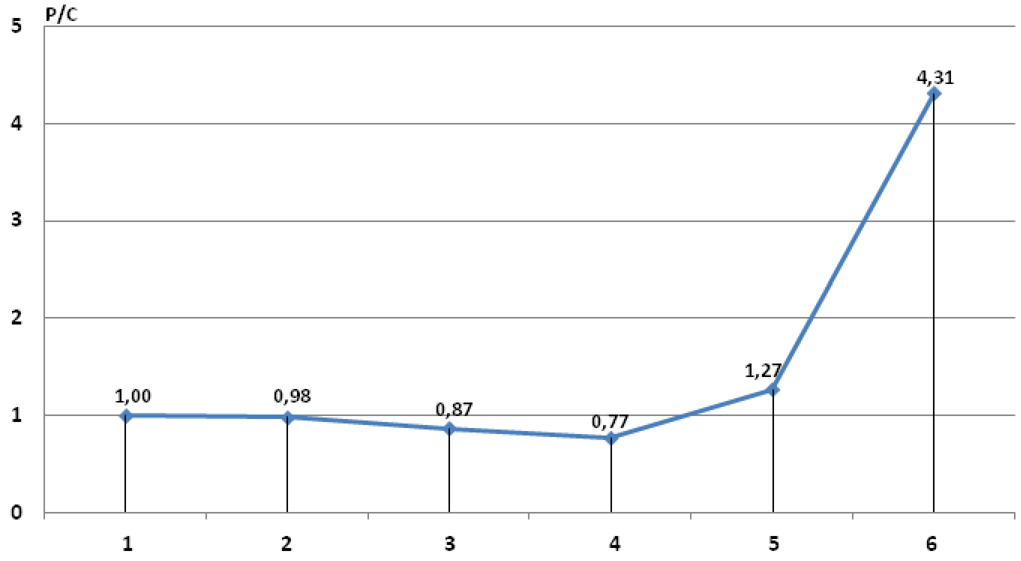

Figure 34. Relative profitableness for the series with five elements in the six analysis models.

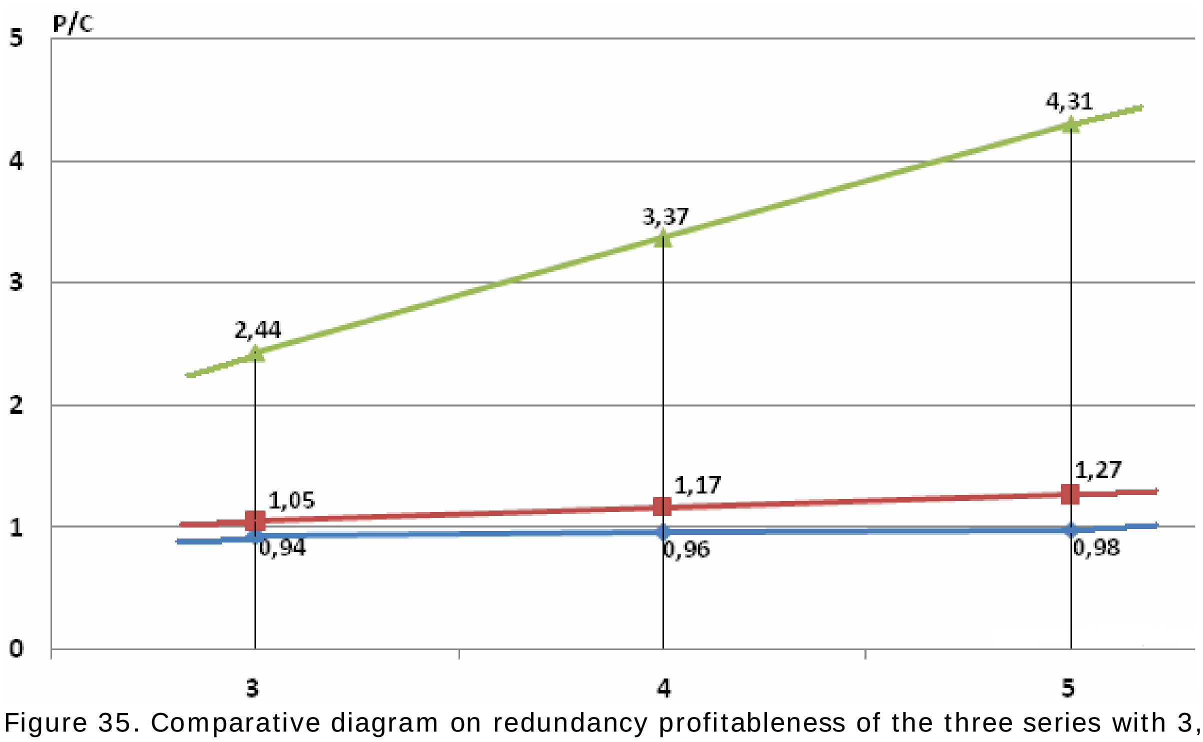

Figure 35. Comparative diagram on redundancy profitableness of the three series with 3, 4 and 5 elements in three topological types of connection: 1) locally-blue line, 2) by twining-red line and 3) globally-green line.

Local parallel connection of one or more redundancies to a single element of the series is not at all profitableness. On the contrary, the profitableness of a topological scheme significantly decreases with the number of redundancies even if in the same time the corresponding durability slowly increases.

Redundancy connection by twining is little efficient namely with 5% to the series with three elements, 17% to the series with four elements, and 27% to the series with five elements. Connection of a single redundancy, of the same value or capacity with the all elements, in parallel with the whole series is miraculous. The profitableness increases in this case with 144%, 237% and 331% respectively.

The above-mentioned outputs about probable durability of critical infrastructures are of particular interest in the plausibility checks according to the *clause 7.4* in ISO 13822:2001. It is the only accurate way to prove that *the limit state of serviceability*, as requested by Eurocode 1 and Romanian Code CR 0-2005, has been fulfilled or not. Moreover, the available analysis tools allow controlling the efficiency of any intervention or alteration necessary to enhance the profitableness of topological models.

Conclusion. The most important parameter in defining the durability of critical infrastructures is the risk factor λ**.** It has a probabilistic nature and therefore it can be never ascertained or found out from a single infrastructure. The risk factor can be only evaluated, designed or accepted for typical infrastructures. In all three cases it has a cost which for critical infrastructures is not at all low. When time is available, by monitoring activities, one obtains useful information. Then, the acquired data should be very carefully processed and adapted to the specific features of each infrastructure. The aim of all maintenance programs devoted to enhance the durability consists in reducing the risk factors. This is why the evaluation of all risks is a major task for the owners of critical infrastructures. The risks are usually originated by natural causes or human errors. In both cases, if properly used, the analysis tools based on the Mathematical Theory of Reliability are of the highest efficiency.

References

Dordea M., Coman N., 2007 [Mankind and its food demands], pp.143-174. In: Petrescu-Mag I. V. (ed.), [Applied Ecology]. Academicpres, Cluj-Napoca. [In Romanian]

Gulvanessian M., 1996 Holický – Designer’s handbook to Eurocode 1. Thomas Telford, London.

Hamaker J. D., (Weawer D. A., ed.) 1982 The Survival of Civilization. Three Problems Threatening Our Existence. Hamaker-Weavers Publishers, USA.

ISO 2394, 1988 General principles on reliability for structures. Geneva, CH.

ISO 13822, 2010 Bases for design of structures – Assessment of existing structures. Geneva CH.

Munich Re Group, Topics Geo. Natural catastrophes 2009. Analyses, assessments, positions. Munich 2010.

Pascu R., 2009 Lessons from the model of climate feedback dynamics. Proceedings of the First International Conference on CIVIL ENGINEERING – TOWARDS A BETTER ENVIRONMENT held on 14-16 June 2009, University of Coimbra, Portugal, pp.133-140.

Petrescu-Mag I. V., 2009 The survival of mankind and human speciation in a complex astrobiological context. ELBA Bioflux **1**(2):23-39.

Sofronie R., 2010 Plausibility checks for built and natural heritage. International Conference on Disaster Management and Cultural Heritage ”Living in Harmony with the Four Elements” 12 - 14 December 2010, Thimphu, Bhutan.

*** http://en.wikipedia.org/wiki/World_population (last view: 12 Jan 2011).

*** http://en.wikipedia.org/wiki/Great_Wall_of_China/ (last view: 12 Jan 2011).

*** http://www.romanianmonasteries.org/bucovina/probota (last view: 12 Jan 2011).

Received: 12 January 2011. Accepted: 25 July 2011. Published online: 25 July 2011.

Authors:

Raluca Pascu, UNESCO Chair, Faculty of Civil and Environmental Engineering, Bucharest, Bd. Marasti 59, Code 011464, Phone/fax: 0213183076, e-mail: ralucaioana.pascu@gmail.com

Ramiro Sofronie, UNESCO Chair, Faculty of Civil and Environmental Engineering, Bucharest, Bd. Marasti 59, Code 011464, Phone/fax: 0213183076, e-mail: ramiro.sofronie@gmail.com

How to cite this article:

Pascu R., Sofronie R., 2011 Durability of critical infrastructures. AES Bioflux **3**(2):167-186.