Stabilization of Interconnected Systems with Decentralized State and/or Output Feedback

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Abstract. This paper investigates the problem of the stabilization of a system (A, B_d) consisting of two interconnected subsystems, with decentralized state or output feedback. Following the initial definition of the global system and of its two subsystems in the state-space, and based on the intercontrollability matrix D(s) of system (A, B_d) and on the kernel U(s) of D(s), an equivalent system {M(s), I_2} defined in the operator domain by an appropriate polynomial matrix description (PMD) is determined. The interconnected system can then stabilized with a suitable local feedback, based on which, a decentralized output feedback can be determined as well.

Keywords: control systems, modeling and simulation interconnected systems, decentralized stabilization, local output feedback.

1 Introduction

Decentralized control has been a control of choice for large-scale systems (consist of many interconnected subsystems) for over four decades. It is computationally efficient to formulate control laws that use only locally available subsystem states or outputs. Such an approach is also economical; since it is easy to implement and can significantly reduce costly communication overhead. Also, when exchange of state information among the subsystems is prohibited, decentralized structure becomes an essential design constraint. Necessary and sufficient conditions, as well as methods and algorithms have been proposed in these four decades, to find decentralized feedback controllers which stabilize the overall system (see (Ikeda,1980), (Sandell, 1978), (Siljak, 1978), (Wang, 1973) and the references therein). In recent years, the problems of decentralized robust stabilization for interconnected uncertain linear systems have been studied by many researchers. Different design approaches have been proposed, such as the Riccati approach (Ge, 1996), (Ugrinovsskii , 1998), the LMI (Linear Matrix Inequality) approach (Liu, 2004), (Souza, 1999), a combination of genetic algorithms and gradient-based optimization (Labibi, 2003), (Patton, 1994).

It is the main purpose of this paper to present the stabilization problem of an interconnected (global) system with decentralized state or output feedback. The interconnected (global) system (A, B_d), consists of two local scalar subsystems,

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under the very general assumptions of the global and the local controllability. It is noted that only the case of two interconnected subsystems is examined, since only then the global system will have no decentralized fixed modes, when local static state-vector feedbacks are applied ((Aderson, 1979), (Caloyiannis, 1982), (Davison, 1983), (Fessas, 1982,1987,1988) and (Wolovich, 1974). Additionally, we assume, without loss of generality (Davison, 1983) that both input channels of system are scalar.

Following the initial definition of the system in the state-space, an equivalent system - defined in the operator domain- is first determined. This is presented in the next section, together with some known results concerning (a) the intercontrollability matrix D(s) of (A, B_d) (b) the kernel U(s) of D(s), (c) the equivalent system {M(s), I_2} in the operator domain, and (d) the stabilization of the interconnected system with linear, local, state-vector feedback (LLSVF), introducing linear programming methods for computing them (Parisses, 1998). In case the values of these feedbacks are considered to be large for practical implementation, an algorithm for designing "optimal" decentralized control can be applied (Parisses, 2006). In section 3, the main result on the stabilizing local output feedbacks, and on a method to design a suitable output matrix C, is presented. As a corollary, the decentralized version of all theses is given. To demonstrate this illustrative example is given, in section 4.

2 Preliminaries

2.1 Form of Matrices A and B_d

We consider the interconnected system (A, B_d) defined by

$$\dot{x} = Ax + B_d u \tag{1}$$

where x is the n-dimensional state of (A, B_d), u is the 2-dimensional input vector, A is the nxn system matrix, and B_d is its nx2 input matrix. Matrices A and B_d admits the following partitioning:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{matrix} \updownarrow n_1 \\ \updownarrow n_2 \end{matrix} \quad \text{and} \quad B_d = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \tag{2}$$

with $n=n_1+n_2$. System (A, B_d) consists of the interconnected n_i-dimensional subsystems (A_{ii}, b_{ii}) -i=1,2- of local state vectors x_1 and x_2, with $x=[x_1' \ x_2']'$, u_1 and u_2 being, respectively, the scalar inputs of these subsystems, with $u=[u_1 \ u_2]'$. We further assume that the global system (A, B_d), as well as its two subsystems (A_{ii}, b_{ii}) -i=1,2- are controllable. In that case, and in order to have some analytical results, subsystems (A_{ii}, b_{ii}) are supposed to be in their companion controllable form (Kailath, 1980)

$$A_{ii} = \begin{bmatrix} 0 & 1 & . & 0 \\ 0 & 0 & . & . \\ 0 & 0 & . & 1 \\ & \mathbf{a}'_{ii} & & \end{bmatrix} \ (n_i x n_i) \ \text{and} \quad b_{ii} = \begin{bmatrix} 0 \\ . \\ 0 \\ 1 \end{bmatrix} \ (n_i x 1) \tag{3}$$

where a_{ii}' denotes the last row elements of A_{ii}. When (A_{ii}, b_{ii}) are in the above form, the $n_i x n_j$ sub matrices A_{ij} assume no particular form; for them we use the notation

$$A_{ij} = \begin{bmatrix} A_{ij}^{o} \\ \mathbf{a}'_{ij} \end{bmatrix} \ (i \neq j) \tag{4}$$

where a_{ij}' denotes the last row elements of A_{ij}, and A_{ij}^{o} the others. It is obvious that when the various sub matrices of (A, B_d) are in the above form, system $(\widehat{A}, \widehat{B}_d)$ is called Canonical Interconnected Form (Fessas, 1982) and is as:

$$\widehat{A} = \left[\begin{array}{ccc|ccc} 0 & 1 & & & A_{12}^{0} & \\ & & . & & & \\ & & 1 & & & \\ & \mathbf{a}'_{11} & & & \mathbf{a}'_{12} & \\ \hline & A_{21}^{0} & & 0 & 1 & \\ & & & & & . \\ & & & & & 1 \\ & \mathbf{a}'_{21} & & & \mathbf{a}'_{22} & \end{array}\right] \quad \widehat{B}_d = \left[\begin{array}{c|c} 0 & \\ . & \mathbf{0} \\ . & \\ 1 & \\ \hline & 0 \\ \mathbf{0} & . \\ & . \\ & 1 \end{array}\right] \tag{5}$$

Finally, with the elements of rows n_1 and $n_1+n_2(=n)$ of A, we form matrix A_m:

$$A_m = \begin{bmatrix} \mathbf{a}'_{11} & \mathbf{a}'_{12} \\ \mathbf{a}'_{21} & \mathbf{a}'_{22} \end{bmatrix} \tag{6}$$

2.2 The intercontrollability matrix D(s) and its kernel

The following (n-2)xn polynomial matrix is the intercontrollability matrix of system $(\widehat{A}, \widehat{B}_d)$:

$$D(s) = \left[\begin{array}{cccc:cccc} s & -1 & & & & & & \\ & \cdot & \cdot & & & -A_{12}^{0} & & \\ & & \cdot & \cdot & & & & \\ & & s & -1 & & & & \\ \hdashline & & & & s & -1 & & \\ & & & & & \cdot & \cdot & \\ & -A_{21}^{0} & & & & & \cdot & \cdot \\ & & & & & & s & -1 \end{array}\right] \quad \textbf{(7)}$$

As the following lemma indicates, D(s) expresses the conditions for the controllability of (A, B_d):

Lemma 2.1: (Caloyiannis, 1982) System (A, B_d) is controllable if and only if rank D(s) = n-2 for all complex numbers s. (Caloyiannis, 1982).

Thus the matrix D(s) of a controllable system is a full-rank matrix. Its kernel U(s) is an nx2 polynomial matrix of rank 2, such that D(s) U(s) = 0. The analytical determination of U(s) is as follows: P is the matrix representing the column permutations of matrix D(s), which brings it to the form of the matrix pencil:

$$\tilde{D}(s) = D(s)\,P = [s\,I_{n-2} - F \mid G] \quad \textbf{(8)}$$

In (8) G is an (n-2)x2 (constant) matrix, consisting of columns $n_1$and $n_1+n_2=n$ of D(s), F is an (n-2)x(n-2) constant matrix, and I_{n-2} is the unity matrix of order n-2. Since D(s) is a full rank matrix, the pair (F,G) is controllable, and can be brought to its Multivariable Controllable Form (MCF) ((Kailath, 1980), (Wolovich, 1974)) $(\hat{F},\hat{G})$ by a similarity transformation T; let d_1, d_2 be the controllability indices of (F,G), S(s) be the associated structure operator, let δ(s) be the characteristic (polynomial) matrix of F, and (in case rank[G]=2) let $\hat{G}_m$ be the 2x2 matrix consisting of rows d_1 and $d_1+d_2=n-2$ of G. The precise form of U(s) is the content of the following lemma:

Lemma 2.2 Let D(s) be the intercontrollability matrix of (A, B_d) as in (7), and suppose that rank[G]=2, for G as in (8). Then the kernel U(s) of D(s) is equal to

$$U(s) = P\left[\begin{array}{c} TS(s) \\ \hdashline -\hat{G}_m^{-1}\delta(s) \end{array}\right] \quad \textbf{(9)}$$

where P, T, S(s), $\hat{G}_m$, and δ(s) are as previously explained.

2.3 An equivalent system defined by a PMD

Consider the interconnected system (A, B_d), with A and B_d as in (5). In that case, the corresponding differential equation in the state space is:

$$\dot{x}(t)=Ax(t)+B_d u(t) \quad \textbf{(10)}$$

In the operator domain, this equation corresponds to the equation

$$(sI - A)\ x(s) = B_d\ u(s) \quad \textbf{(11)}$$

this, in its turn, reduces to the equations:

$$D(s)\ x(s) = 0 \quad \textbf{(12a)}$$

and

$$(sE - A_m)\ x(s) = u(s) \quad \textbf{(12b)}$$

In these equations, D(s) is as in (7), E is a 2xn (constant) matrix, of the form: E = diag$\{e_1'\ e_2'\}$, the n_i-dimensional vector e_i being equal to : e_i = [0...0 1]' -for i=1,2-, and A_m is the matrix defined in (6). From (12a) it follows that x(s) must satisfy the relation:

$$x(s) = U(s)\ \xi(s) \quad \textbf{(13)}$$

where U(s) is the kernel of D(s), and ξ(s) is any two-dimensional vector. It follows that ξ(s) must satisfy the equation:

$$M(s)\ \xi(s) = u(s) \quad \textbf{(14)}$$

The matrix M(s) appearing in (14) is termed Characteristic Matrix of the interconnected system (A, B_d) (Fessas, 1982) and is defined by the relation:

$$M(s) = (sE - A_m)\ U(s). \quad \textbf{(15)}$$

The three systems defined respectively (i) in the state space by the pair of matrices (A, B_d), (ii) in the operator domain by {sI-A, B_d}, and (iii) by the polynomial matrix description (PMD):

$$M\ (D)\ \xi\ (t) = u\ (t) \quad \textbf{(16a)}$$

$$X\ (t) = U\ (D)\ \xi\ (t) \quad \textbf{(16b)}$$

are equivalents (Chen, 1984), (Fessas, 1987), (Kailath, 1980). It is noted that in (16) ξ(t) is the pseudo state vector of the system, and is related to the state vector x(t) of (A, B_d), by the relation

$$x(t) = U(D)\ \xi(t) \quad \textbf{(17)}$$

(in the relations (16), (17), the symbol D denotes the differential operator d/dt).

2.4 Stabilizability with local state-vector feedback

We present analytically Theorem 2.1, on the stabilizability of the interconnected system (A, B_d) with LLSVF, as well as a result, which is needed in the proof of it.

Lemma 2.3 : Let h(s) be a polynomial of the form: h(s)=r(s)p(s)+q(s), for which the following assumptions hold: (i) The polynomials r(s), p(s), q(s) are monic (ii) r(s) is arbitrary, (iii) degree r(s)p(s) > degree q(s) (iv) p(s) is a stable polynomial. Then, the arbitrary polynomial r(s) can be chosen so, that h(s) is stable (Seser, 1978).

Theorem 2.1: Consider the interconnected system (A, B_d) as in (1), and suppose that the global system (A, B_d), and the local ones (A_{ii}, b_{ii}) -i=1,2- are controllable.

Then, there exists a static LLSVF of the form $u=K_d x$, so that the resulting closed-loop system is stable.

Proof: For the proof we consider the equivalent system $\{M(s), I_2\}$ and examine the stability of the polynomial matrix $M_d(s)=(sE-A_m-K_d)U(s)$. We assume that the feedback matrix K_d has the form:

$$K_d=\left[\begin{array}{cccc|cccc}\alpha_1 & . & . & \alpha_{n_1} & 0 & . & . & 0 \\ \hline 0 & . & . & 0 & \beta_1 & . & . & \beta_{n_2}\end{array}\right]=\begin{bmatrix}\boldsymbol{\alpha}' & 0 \\ 0 & \boldsymbol{\beta}'\end{bmatrix} \qquad \textbf{(18)}$$

where α_i (i=1,...,n_1), and β_j (j=1,...,n_2) are some unknown, real numbers. We shall deal with the case where rank [G]=2, which is the usual one for the matrix G. Then the matrix $M_d(s)$ takes the form:

$$M_d(s)=(sE-A_m-K_d)U(s)=(sE-A_m-K_d)\,P\left[\begin{array}{c}TS(s) \\ \hline -\hat{G}_m^{-1}\delta(s)\end{array}\right]=$$

$$=\left[\begin{array}{c|c}-(s-a_{n1}-a_{1,n1})[\mathbf{11}]-a(s)+a_{1,n}[\mathbf{21}] & -(s-\alpha_{n1}-a_{1,n1})[\mathbf{12}]-\alpha_1(s)+a_{1,n}[\mathbf{22}] \\ \hline -(s-\beta_{n2}-a_{2,n1})[\mathbf{21}]-\beta_1(s)+a_{2,n1}[11] & -(s-\beta_{n2}-a_{2,n})[\mathbf{22}]-\beta(s)+a_{2,n1}[\mathbf{12}]\end{array}\right]=$$

$$=\begin{bmatrix}M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s)\end{bmatrix} \qquad \textbf{(19)}$$

where

$$\alpha(s)=[\alpha_1+a_{1,1}\cdots\alpha_{n_1-1}+a_{1,n_1-1}\;\; a_{1,n_1+1}\cdots a_{1,n-1}]TS_1(s)$$

$$\alpha_1(s)=[\alpha_1+a_{1,1}\cdots\alpha_{n_1-1}+a_{1,n_1-1}\;\; a_{1,n_1+1}\cdots a_{1,n-1}]TS_2(s)$$

$$\beta(s)=[a_{2,1}\cdots a_{2,n_1-1}\;\; \beta_1+a_{2,n_1+1}\cdots\beta_{n_2-1}+a_{2,n-1}]TS_2(s)$$

$$\beta_1(s)=[a_{2,1}\cdots a_{2,n_1-1}\;\; \beta_1+a_{2,n_1+1}\cdots\beta_{n_2-1}+a_{2,n-1}]TS_1(s)$$

are scalar polynomials, not monic,

$$TS_1(s) = T\,[\,1\ s\ \ldots\ s^{d_1-1}\ 0\ \ldots\ 0\,]'$$

$$TS_2(s) = T\,[\,0\ \ldots\ 0\ 1\ s\ \ldots\ s^{d_2-1}\,]'$$

(i.e., $TS(s)=[TS_1(s)\ TS_2(s)]$,and [**ij**] -for i,j=1,2- are the entries of the polynomial matrix $\hat{G}_m^{-1}\delta(s)$. Then the matrix in (19) is equivalent to the following matrix:

$$M_d'(s)=\begin{bmatrix}M_{11}(s)+M_{21}(s) & M_{12}(s)+M_{22}(s) \\ M_{21}(s) & M_{22}(s)\end{bmatrix} \qquad \textbf{(20)}$$

$h(s)=\det M_d'(s)=\{M_{11}(s)+M_{21}(s)\}M_{22}(s)-\{M_{12}(s)+M_{22}(s)\}M_{21}(s)=$

$=\{M_{11}(s)+M_{21}(s)\}\{-\beta(s)-(s-\beta_{n2}-a_{2,n})[\mathbf{22}]+a_{2,n1}[\mathbf{12}]\}-\{M_{12}(s)+M_{22}(s)\}M_{21}(s)=$

$=-[\mathbf{22}]\{M_{11}(s)+M_{21}(s)\}\,(s-\beta_{n2}-a_{2,n})-\{M_{12}(s)+M_{22}(s)\}M_{21}(s)+\{M_{11}(s)+M_{21}(s)\}$

$\{-\beta(s)+a_{2,n1}[\mathbf{12}]\}=r(s)p(s)+q(s)$ **(21)**

The determinant of this matrix is actually a monic polynomial of degree n, by identifying r(s) as the polynomial $-[\mathbf{22}][M_{11}(s)+M_{21}(s)]$, which is of degree (n-1), arbitrary and monic, p(s) as the polynomial $(s-\beta_{n2}-a_{2,n})$, which is stable by choice of β_{n2}, and q(s) as the polynomial $-\{M_{12}(s)+M_{22}(s)\}M_{21}(s)+\{M_{11}(s)+M_{21}(s)\}\{-\beta(s)+a_{2,n1}[\mathbf{12}]\}$, of degree (n-1). Then, according to lemma, the arbitrary polynomial r(s) can be chosen so that the polynomial h(s) is stable. Q.E.D.

This proof is completed with an iterative method (Parisses, 1998), in order to compute the feedback coefficients. The central idea is to compute the feedback parameters by solving a linear programming problem (Luenberger, 1984) corresponding to choosing positive the coefficients of the polynomials that should be stable. A set of such polynomials (with positive coefficients) is generated. They are then examined whether they are stable or not.

ALGORITHM

Step1 Choose the feedback parameter $\beta_{n2}+\alpha_{2,n}<0$ so that a stable p(s) results.

Step2 Write the polynomial r(s) in the following form:

$r(s)=s^{n-1}+k\rho(s)=s^{n-1}+k(s^{n-2}+k_1 s^{n-3}+...+k_{n-2})$.

By viewing the degrees of the polynomials α(s) and β(s), it is seen that k-the leading coefficient of the polynomial ρ(s)- contains only the parameters β_{n2} and α_{n1}. It follows that by giving a value to k, we can also compute α_{n1}.

Step3 Form n-2 inequalities with the n-2 unknown feedback parameters, by setting positive the coefficients k_i of the polynomial ρ(s) ($k_i>0$, for i=1,n-2).

Step4 Solve the linear programming problem, by putting an objective function with unity weighting coefficients, and find all feedback parameters α_i and β_j.

Step5 Evaluate the polynomial ρ(s), and check if it is stable. If it is not, go back to Step 1, and select another β_{n2}.

Step6 Evaluate the polynomial r(s), and check if it is stable. If it is not, go back to Step 2, and select another k.

Step7 Evaluate the polynomial h(s), and check if it is stable. If it is not, go back to Step 1, and select another β_{n2}.

Step8 The feedback matrix K_d can be evaluated from steps 1, 2, and 4.

END OF THE ALGORITHM

3 Main Result

Theorem 3.1 Consider the interconnected system (A, B_d) as in (1), under the usual assumptions of the global and the local controllability. Then, this system can stabilized with the feedback u=Ly, where the output feedback matrix L is arbitrary, and the output matrix C is: $C=L^{-1}K_d$, matrix K_d being the feedback stabilizing matrix.

Proof: Since system (A, B_d) satisfies the assumptions of the global and the local controllability, there exists a local feedback stabilizing matrix K_d, such that $A+B_dK_d$ is stable. According to lemma 2.1 of (Fessas, 1994), system (A, B_d, C) can stabilized with the output feedback u=Ly, when the output matrix C is given by the relation $C=L^{-1}K_d$.

Remark 3.1 It is remarked that, while the 2x2 output matrix L is arbitrary, it is the 2xn matrix C that takes care of the stabilization. As an extreme case consider $L=I_2$ (the unity matrix); it is follows that the output matrix C is identical to the stabilizing local state feedback matrix K_d.

Corollary 3.1 We consider matrix L as a diagonal L_d matrix (corresponding to the control with local feedbacks). It follows that the output matrix C is also block-diagonal $C_d=L_d^{-1}K_d$ corresponding, thus, to the case where the measurements are also decentralized.

4 An illustrative example

The controllable system (A, B_d) is:

$$A=\left[\begin{array}{cc:cc} 1 & 2 & -1 & 0 \\ -4 & 0 & 1 & 2 \\ \hdashline -3 & -2 & 2 & -1 \\ 5 & 0 & 3 & 4 \end{array}\right] \qquad B_d=\left[\begin{array}{c:c} 1 & 0 \\ 1 & 0 \\ \hdashline 0 & 1 \\ 0 & 1 \end{array}\right]$$

The system is unstable, since the eigenvalues of A are: {0.315±2.732j, 3.185± 1.511j}, and it is asked to be stabilized by the d-control $u=K_dx$. Subsystems A_{ii} -i=1,2- are transformed into their companion forms, by the transformation matrices:

$$T_1=\begin{bmatrix} 2 & 1 \\ -5 & 1 \end{bmatrix} \qquad T_2=\begin{bmatrix} -5 & 1 \\ 1 & 1 \end{bmatrix}$$

while matrix $\hat{A}$, as in (5), is:

$$\hat{A}=\left[\begin{array}{cc:cc} 0 & 1 & 1.143 & -0.571 \\ -8 & 1 & 2.714 & 0.143 \\ \hdashline 1 & 1.667 & 0 & 1 \\ 9 & 3.333 & -11 & 6 \end{array}\right]$$

The intercontrollability matrix D(s), of system $(\hat{A}, \hat{B}_d)$ is:

$$D(s)=\begin{bmatrix} s & -1 & -1.143 & 0.571 \\ -1 & -1.667 & s & -1 \end{bmatrix}$$

It follows that system (F, G) is given by:

$$F=\begin{bmatrix}0 & 1.143\\ 1 & 0\end{bmatrix} \qquad G=\begin{bmatrix}-1 & 0.571\\ -1.667 & -1\end{bmatrix}$$

The controllability indices d_1 and d_2 of (F, G) are: $d_1=1$, $d_2=1$. Obviously, rank [G]=2. The canonical form of matrix F is:

$$\hat{F}=\begin{bmatrix}1.268 & 0.418\\ -1.114 & -1.268\end{bmatrix}$$

The kernel U(s), of D(s), is:

$$U(s)=\begin{bmatrix}-1 & 0.571\\ -s+1.268 & 0.418\\ -1.667 & -1\\ 1.114 & -s-1.268\end{bmatrix}$$

At this point begins the search for stable polynomials, by applying simultaneously the linear programming method, as described by the algorithm. We use the same notation as in the text, and give the final results: $\beta_{n2}=-8.0$, $\alpha_{n1}=-28.22$. Polynomial $\rho(s)=s^2+3.25s+2.60$ (roots of $\rho(s)$:-1.84, -1.41). Polynomial $r(s)=s^3+25.00s^2+81.35s+65.00$ (roots of r(s) : -21.33, -2.40, -1.27), and finally $h(s) = s^4+29.22s^3+122.44s^2+1108.14s+1624.01$. The roots of this polynomial are the numbers {-26.01, -0.75±6.09j, -1.66}, which are the eigenvalues of the closed-loop system, i.e., of system $\widehat{A}+\widehat{B}_d K_d$. The matrix of the feedback parameters is:

$$K_d=\left[\begin{array}{cc:cc}-25 & -28.22 & 0 & 0\\ \hdashline 0 & 0 & -25 & -8\end{array}\right]$$

It is remarked that the above values of K_d are in the transformed system of coordinates (used to apply the method based on the equivalent system defined by a PMD). For a given matrix

$$L=\begin{bmatrix}10 & 30\\ 20 & 50\end{bmatrix}$$

the output matrix C is

$$C=\left[\begin{array}{cc:cc}12.5 & 14.11 & -7.5 & -2.4\\ \hdashline -5 & -5.644 & 2.5 & 0.8\end{array}\right].$$

If we suppose, as corollary 3.1 diagonal L

$$L_d=\begin{bmatrix}10 & 0\\ 0 & 50\end{bmatrix}$$

the corresponding matrix C_d is block-diagonal, where the measurements are indeed decentralized.

$$C_d=\left[\begin{array}{cc:cc}-2.5 & -2.822 & 0 & 0\\ \hdashline 0 & 0 & -0.5 & -0.16\end{array}\right].$$

5 Conclusion

In this paper we considered the stabilization of a global system (A, B_d), resulting from the interconnection of subsystems (A_{ii}, b_{ii}) -i=1,2-, with decentralized state and/or output feedback control. We studied initially the problem of the stabilization of (A, B_d) with linear, static feedback of the local state-vectors, under the weak conditions of the global and the local controllability. Although the problem was defined in the state-space, it was transformed into the frequency domain and studied therein. The existence of a local, feedback stabilizing matrix was formally proven and it is completed by a numerical procedure -based on linear programming methods- for the numerical computation of the feedback parameters (K_d). It is supposed the output feedback matrix L is arbitrary, and one wishes to determine the appropriate output matrix C which 'realizes' the decentralized feedback $u=K_dx$, by the matrix $C=L^{-1}K_d$. That corresponds to what (Zheng , 1989) refers to as 'the designer's possibility to choose the output matrix C'.

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