A Method for Deciding Node Densities in Non-Uniform Deployment of Wireless Sensors

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***Abstract*—In a wireless sensor network, if the sensors are deployed uniformly across the network, they experience different traffic intensities, thereby, different energy depletion rates depending on their locations. Usually, the sensors near the sink tend to deplete their energy sooner; when enough of them exhaust their energy, they leave holes in the network, causing the remaining nodes to be disconnected from the sink. One of the solutions to this energy-hole problem is to deploy the sensors non-uniformly. This paper describes a method for deciding the sensor deployment densities so as to equalize the energy consumption rates of all nodes. The method is general and can be applied to other objectives and constraints.**

***Index Terms*—Wireless Sensor Networks; Node Deployment; Energy Holes; Node Density; Energy Management**

I. Introduction

Wireless sensor networks have diverse applications such as environmental monitoring (e.g., vehicular traffic, wild life habitat, bridge or earthquake monitoring) and battlefield surveillance. A sensor network can use multi-hop routing to deliver the collected information to the collection center, or the sink. The sensor nodes typically face severe energy constraints. They usually have limited on-board batteries and are often deployed in harsh environment where human operators cannot access them easily to replace the batteries. As a result, much of the research on sensor networks has focused on how to prolong their lifetime [1] [2] [3] [4] [5] [6], for instance, by using energy-efficient routing strategies. The lifetime of a sensor network has several definitions in the literature. One of the most popular definitions has it as the interval of time from the system startup until the first node exhausts its energy, i.e., the shortest lifetime of all the nodes.

Regardless of the energy-saving strategies used, sensor networks often experience unbalanced traffic distribution because the multi-hop traffic pattern is usually many-to-one [7], [8], [9]. The traffic transmitted by each sensor node typically includes both self-generated and relayed traffic. Since the entire network traffic flows toward the sink, the nodes closer to the sink tend to experience more traffic. As a result, their energy consumption rates tend to be higher than the nodes far away from the sink. This causes the nodes closer to the sink to deplete their energy sooner, leaving a hole near the sink and partitioning the whole network while many remaining nodes still have much energy. The phenomenon is called the *energy hole problem* [9], [10]. In [8], the authors observe that when the nodes one hop away from the sink exhaust all their energy, the remaining nodes have used only 7% of their energy on average.

On closer examination, energy holes are most often observed in networks where homogeneous sensor nodes are uniformly deployed. If we allow non-uniform deployment of the sensor nodes, by carefully planning the number of nodes in different places of the network, we can prevent the sensor nodes near the sink from depleting their energy faster than others, and hence, resolve the energy hole problem. This solution may require adding more nodes than what is needed for coverage in some parts of the network, which means a higher cost. Hence, the solution makes sense in situations where inexpensive sensors can be mass-produced or having a longer network lifetime outweighs the cost of the extra sensors. Recent advances in micro-electro-mechanical and integration technologies make the first situation more and more likely to occur.

This paper contributes to the research area that seeks to extend network lifetime by deploying the sensors non-uniformly. The main question to be addressed is how many nodes per unit area (i.e., the *node density*) should be deployed in different parts of the sensor field in order to achieve a prescribed lifetime-cost objective. The main result of the paper is a mathematical method for computing the node densities.

The method will be illustrated using an example which has the particular objective of equalizing the energy consumption rates of the sensor nodes throughout the network, while minimizing the total number of nodes deployed. We will show how to derive the location-dependent node densities in this case. The result is that all nodes will exhaust their energy at the same time, and hence, energy holes will not emerge. The method itself is intended to be general. It can be adapted for other lifetime-cost objectives and other constraints including routing schemes not included in this paper.

Among the studies about the energy hole problem or the uneven traffic distribution problem ([8], [11], [4], [7], [9], [10]), [10] is the most similar to our work in its goal of obtaining a balanced energy consumption rate everywhere by non-uniform node deployment. In that work, the sensor field is divided into several concentric rings around the sink. The authors give a heuristic routing scheme that achieves an equal energy dissipation rate in all rings except the outmost one, provided the number of nodes increases geometrically from the outmost ring inward. However, their models of routing, the sensor field and energy consumption are significantly different from ours, and consequently, the problems of determining the node densities are quite different, with ours being more general. Some of the important differences are:

1) In [10], the sensor nodes can send data only to the nodes

in the neighboring ring. In contrast, we model a wide class of routing schemes. In each routing scheme, the nodes can send data to different inner rings at different probabilities.

2) In [10], the nodes generate data at the same constant rate. We consider a family of data generate rates, which are functions of the node densities.
3) Our energy consumption model is more general: The required transmission power of a node is a function of the transmission distance to the receiver.

The rest of the paper is organized as follows. In Section II, we show how to compute the node densities required to equalize the energy consumption rate. There, we consider general energy consumption, traffic generation and routing models. In Section III, we show how to compute the node densities for several concrete and important cases. In Section IV, we show experimental results to demonstrate the validity of our modeling approach and analytical method. The conclusions are given in Section V.

II. Deciding Node Densities - General Case

In this section, we show how to compute the node densities required to equalize the energy consumption rates of all nodes in the network. We consider a family of data generation models, a family of routing schemes, and a general energy consumption model. We give a complete solution for the case of density-independent routing.

A. Sensor Network and Energy Consumption Models

The sensor field in the shape of a disk is shown in Figure 1 with the sink at the center. The disk is divided into concentric rings having the same width. In each ring, the nodes are uniformly spread out and the node density is a constant. We only need to compute a finite number of node densities, one for each ring. We consider the situation where the communication capability of the nodes is limited so that multi-hop routing is necessary to transfer data to the sink. The routing rule is specified at the granularity of the rings.

Such ring-based model of the sensor network is typical (see [8], [10]). It represents a simplification; but it can also be close to reality for a number of practical cases, for instance, when the network is explicitly constructed this way. For us, it mainly serves to explore and illustrate the basic ideas of the method and to make numerical experiments easier. The work on highly general sensor network models is ongoing.

We introduce some definitions and notations.

- n: the total number of rings.
- R_j: ring j, $0 \leq j \leq n$. We index the rings in the direction away from the center of the disk. For convenience, R_0 refers to the center of the disk where the sink is. R_1 is also special; it is a disk.
- w: the width of each ring, $R_1, \cdots, R_n$. For R_1, w is its radius.
- ρ_j: the node density of ring j, $1 \leq j \leq n$.
- S_j, C_j and G_j: the rates of the locally generated traffic, of the relayed traffic, and of the total (outgoing) traffic, respectively, of a typical node in ring j, $1 \leq j \leq n$.
- P_j: the energy consumption rate of a typical node in ring j, $1 \leq j \leq n$.

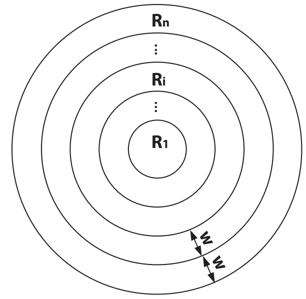


Fig. 1. Sensor field model

The energy consumption model of the sensor nodes affects the final density of each ring. We adopt the energy consumption model of [4]. The required transmission energy to send one unit of data to a node at a distance d away from the sender is given by $E_t(d) = \gamma + \beta d^{\alpha}$, where γ is the required energy to operate the transceiver circuitry, β is a parameter determined by the environment, and α is the so-called *path loss exponent*. Usually, α is between 2 to 6 depending on the operating environment. To account for the situations where the transmitters cannot vary the transmission power due to technological constraints, we widen its range to between 0 and 6. Some amount of energy is also required to receive a unit of data, and this amount is denoted by $E_r(\cdot) = \gamma$. In this model, the receiving energy requirement does not depend on the distance from the transmitter.

The maximum transmission range of a sensor node is also an important parameter. We assume the range is l rings, $1 \leq l \leq n$. That is, a sensor node can transmit data up to l rings away without relaying. If the maximum range is equal to n, then every sensor node is able to send data to the sink directly. This is the assumption of [8], which is more restricted than our case.

B. Traffic and Routing Models

We assume flow conservation at every sensor node: A node cannot buffer an infinite amount of data, and, after the traffic is generated, there is no further in-network processing that may reduce or increase the traffic rate at the node. The data transmission rate at a typical node in ring j, denoted by G_j, can be expressed as

$$G_j = S_j + C_j, \tag{1}$$

where S_j is the rate of the locally generated traffic, and C_j is the rate of the traffic to be relayed by the node.

1) Rate of Locally Generated Traffic (S_j): Let us assume that a certain amount of data rate is needed to monitor a unit of area and the rate is a constant value of K throughout the sensor field. This is the inherent data rate needed for reporting events or conditions about a unit of area. There are many possibilities regarding how the inherent data rate affects the actual traffic rate generated by each nearby sensor node and we will consider several of them.

1) The system has local coordination among the nearby sensors, which can reduce the amount of traffic generated. Specifically, for ring j, the rate of the locally generated traffic at a typical node is $S_j(\rho_j) = K/\rho_j$.
2) Another possibility is the complete lack of local coordination among the nodes and the nearby nodes all report the same events to the sink. In this case, the rate of the traffic generated by any node is $S_j(\rho_j) = K$.
3) The sensor nodes coordinate with each other to reduce the generated traffic. But the coordination is not perfect and there is still some amount of excess traffic. We can model this situation in two ways. The first is to set $S_j(\rho_j) = K/\rho_j^a$ for some constant a, $0 < a < 1$. The amount of data generated per unit area is $K\rho_j^{(1-a)}$, which is more than K (we will see later that we always have $\rho_j \geq 1$ for all j). The constant a is chosen to match the actual amount of generated traffic. The second way is to set $S_j(\rho_j) = \eta K/\rho_j$ for some constant $\eta \geq 1$. The amount of data generated per unit area is ηK. This represents a trivial extension to case 1) above and we will not consider it further.

To summarize, the rate of locally generated traffic by a typical node in ring j is

$$S_j(\rho_j) = K/\rho_j^a, \tag{2}$$

where $0 \leq a \leq 1$.

2) Routing: The rate of the relayed traffic at a node depends on the routing scheme used by the network. We consider the class of routing schemes that can be captured by the following probabilistic description. For each pair of rings k and j, $0 \leq j < k$, let

$$F_k(j) = \{\text{the probability that a node in ring } k \text{ selects a node in ring } j \text{ as its next-hop neighbor for data delivery}\}.$$

Note that the description of the routing is at the granularity of the rings. All nodes in the same ring have the same probability distribution. Each particular matrix $(F_k(j))$ describes a routing scheme. The description is precise if a routing scheme operates exactly according to the definition of $F_k(j)$. We will later consider several concrete examples. In more general cases, the description by $(F_k(j))$ can be thought as a model for a practical routing scheme and it approximately captures how a node chooses its next-hop neighbor for data delivery in the practical routing scheme. Many real routing schemes may be modeled this way.

3) Rate of Relayed Traffic (C_j): With the routing given by $(F_k(j))$, we proceed to derive the relationship satisfied by the relayed traffic. The total rate of traffic that all nodes in ring k transmit directly to ring j, $1 \leq j < k$, is $\rho_k \pi(2k-1)w^2 G_k F_k(j)$. Therefore, the average rate of the traffic contributed by ring k to a typical node in ring j is

$$\frac{\rho_k \pi(2k-1)w^2 G_k F_k(j)}{\rho_j \pi(2j-1)w^2} = \frac{\rho_k(2k-1)G_k F_k(j)}{\rho_j(2j-1)}.$$

For each j, $1 \leq j < n$, the rate of the relayed traffic at a typical node in ring j is the sum of the above quantity over all rings outside ring j that can reach ring j. That is,

$$C_j = \frac{\sum_{k=j+1}^{\min(n,j+l)} \rho_k(2k-1)G_k F_k(j)}{\rho_j(2j-1)}. \tag{3}$$

Note that, in $\min(n, j+l)$, ring n is the outmost ring and l is the maximum range.

Now we can get the total data transmission rate for a node in ring j, $1 \leq j < n$. By (1) and (3), we have

$$G_j = S_j(\rho_j) + \frac{\sum_{k=j+1}^{\min(n,j+l)} \rho_k(2k-1)G_k F_k(j)}{\rho_j(2j-1)}. \tag{4}$$

For the outmost ring n, since $C_n = 0$, we have

$$G_n = S_n(\rho_n). \tag{5}$$

For now, we consider ρ_n as a given parameter. We will discuss how to determine ρ_n later.

C. Power Consumption Rate of a Node

Consider the energy consumption rate of a typical node in ring j, denoted by P_j. P_j depends on the energy consumption model discussed in Section II-A. In the energy model, the energy required to transmit a unit of data is a function of the transmission distance. Continuing to pursue the possibilities for simplification offered by the ring model, we make the approximation that the distance between a pair of nodes is proportional to the number of rings separating them. The approximation will be quite accurate if the next-hop neighbor that each node selects is on the line from the node to the sink, or in the vicinity of the line. Any shortest-path routing or minimum-energy routing algorithms, which are the most popular algorithms, will favor such a neighbor. Hence, the approximation is relevant to typical systems.

The power used for transmission by a node in ring j toward ring i is denoted by $P_j^t(i)$. This is the energy consumed per unit of time to transfer the portion of the node's data directed to the nodes in ring i[1]. We have, for $1 \leq j \leq n$ and $(j-l)_+ \leq i < j$,

$$P_j^t(i) = (\gamma + \beta(j-i)^\alpha w^\alpha) G_j F_j(i).$$

The overall transmission power used by a node in ring j is given by[2]

$$\begin{aligned} P_j^t &= \sum_{i=(j-l)_+}^{j-1} P_j^t(i) \\ &= \gamma G_j + \beta w^\alpha G_j \sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i). \end{aligned} \tag{6}$$

The power used for receiving data by a node in ring j is[3],

$$P_j^r = \gamma C_j = \gamma G_j - \gamma S_j(\rho_j). \tag{7}$$

[1]Here, the probability $F_j(i)$ is interpreted as the proportion of the data at a fixed node in ring j that is transmitted to some nodes in ring i. Throughout, we will use the two interpretations, probability or proportion, interchangeably depending on the need.

[2]For a real number b, we define $(b)_+ \triangleq \max(b, 0)$.

[3]Note that since $C_n = 0$, $P_n^r = 0$.

Hence, the power consumption of a node in ring j is, for $1 \le j \le n$,

$$P_j = P_j^t + P_j^r = 2\gamma G_j + \beta w^\alpha G_j \sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i) - \gamma S_j(\rho_j). \quad (8)$$

D. Problem Formulation

We make the minor assumption that the sensors have identical initial energy endowment. Our objective is to choose node densities $(\rho_i)_{1\le i\le n}$ so as to equalize the energy consumption rate of all nodes, i.e., to have $P_j = P_n$ for all $1 \le j < n$. If this can be achieved, all the sensor nodes will have identical lifetime. There is also a minimum density requirement, i.e., $\rho_i \ge \Theta$ for all i, where Θ is a positive constant. In its present form, the problem may have multiple solutions. If that is the case, we take the one with the smallest total number of sensors, $\sum_{i=1}^{n} \rho_i A_i \le \Lambda$, where each constant A_i is the area of ring i. Putting it together, we have the following optimization problem, called **(EP)**.

$$(\mathbf{EP}) \qquad \min \sum_{i=1}^{n} \rho_i A_i \quad (9)$$

$$\text{subject to} \quad P_j = P_n, \quad \text{for all } j = 1, 2, \ldots, n-1 \quad (10)$$

$$\rho_j \ge \Theta, \quad \text{for all } j = 1, 2, \ldots, n. \quad (11)$$

The above problem is less general than a lifetime maximization problem, which is to maximize the shortest lifetime of all sensor nodes subject to limit on the total number of sensor nodes. However, in important special cases (e.g., when $S(\rho_j) = K/\rho_j$), the two problems are equivalent in the sense that the lifetime maximization problem has an optimal solution in which all sensor nodes have identical lifetime (equivalently, the nodes have identical energy consumption rates). In the cases where the two problems are different, finding simple solutions for the lifetime maximization problem is more challenging. The effort to solve that problem on a very general network model is ongoing. Meanwhile, the paper is restricted to the problem of equalizing the energy consumption rates.

With $P_j = P_n$, G_j can be written as follows, for $1 \le j < n$,

$$G_j = \frac{P_n + \gamma S_j(\rho_j)}{2\gamma + \beta w^\alpha \sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i)}. \quad (12)$$

E. Case of Density-Independent Routing

In this case, the routing $(F_k(j))_{k,j}$ is independent of the node densities $(\rho_k)_k$ everywhere. By (5) and (8), P_n depends only on ρ_n. Since ρ_n is considered a constant, P_n is also a constant. Then, from (12), G_j can be viewed as a function of ρ_j. By re-arranging (4), we get

$$\rho_j(G_j(\rho_j) - S_j(\rho_j)) = \frac{\sum_{k=j+1}^{\min(n,j+l)} \rho_k (2k-1) G_k(\rho_k) F_k(j)}{2j-1}. \quad (13)$$

The left hand side of (13) depends on ρ_j.

Hence, if we are given ρ_k and G_k for $k \ge j+1$, we can compute ρ_j by solving a single variable equation. Once ρ_j is known, we can compute $G_j(\rho_j)$ by (12). The additional knowledge of ρ_j and $G_j(\rho_j)$ sets the stage to solve ρ_{j-1} next. The procedure repeats until ρ_1 is solved.

In the following, the details about how to solve ρ_j are explained for the three traffic generation models. Let

$$B_1 \triangleq 2\gamma + \beta w^\alpha \sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i) \quad (14)$$

$$B_2 \triangleq \frac{\sum_{k=j+1}^{\min(n,j+l)} \rho_k (2k-1) G_k F_k(j)}{2j-1}. \quad (15)$$

Since the routing is independent of the densities and it is given, B_1 is a known value for the fixed j. Given ρ_k and G_k for $k \ge j+1$, B_2 is also known. Then, from (12) and (13), we have

$$\rho_j P_n + (\gamma - B_1)\rho_j S_j(\rho_j) = B_1 B_2. \quad (16)$$

1) $S_j(\rho_j) = K/\rho_j$:

$$\rho_j = \frac{B_1 B_2 - K(\gamma - B_1)}{P_n}. \quad (17)$$

Note that $\gamma - B_1 < 0$ and $B_2 > 0$. Hence, positive solutions for ρ_j's exist.

2) $S_j(\rho_j) = K$: In this case, by (6) and the fact $P_n^r = 0$, we have $P_n = \gamma K + \beta w^\alpha K \sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i)$. Then, $P_n + K(\gamma - B_1) = 0$. Hence, the left hand side of (16) is $\rho_j(P_n + K(\gamma - B_1)) = 0$. There is no solution to ρ_j. This means that it is impossible to have $P_j = P_n$ for $1 \le j < n$. The result is not surprising. The locally generated traffic rate is K for every node in ring j. In general, each node also receives and relays some traffic from the outer rings. The total traffic rate from a node in ring j exceeds K. On the other hand, the traffic rate from a node in ring n is K. There is no way to equalize the energy consumption rates of the two nodes. In this case, the lifetime maximization problem is a more suitable formulation. We will not pursue that problem in this paper.

3) $S_j(\rho_j) = K/\rho_j^a$, $(0 < a < 1)$:

$$\rho_j P_n + (\gamma - B_1) K \rho_j^{1-a} = B_1 B_2. \quad (18)$$

In general, there is no closed form expression for the solution of ρ_j. But, this single-variable equation can be easily solved numerically. Consider the left-hand side function $g(\rho_j) \triangleq \rho_j P_n + (\gamma - B_1) K \rho_j^{1-a}$. Note that $g(0) = 0$ and $g(\rho_j)$ eventually increases to infinity. The derivative of the function is $g'(\rho_j) = P_n + (\gamma - B_1)K(1-a)\rho_j^{-a}$, which increases from $-\infty$ to P_n on the interval $(0, \infty)$. Hence, on $[0, \infty)$, $g(\rho_j)$ starts at 0, decreases, and then increases to infinity. Since $B_1 B_2 > 0$, there is always a unique solution to the equation $g(\rho_j) = B_1 B_2$. One of the ways to find the solution is binary search.

To summarize, under a given $\rho_n > 0$, ρ_j can be computed iteratively from $j = n-1$ down to $j = 1$. For the case of $a = 1$, the formula (17) can be used directly; for the case $0 < a < 1$, one can solve (18) by binary search (or any other general numerical procedure for solving a single-variable equation). The algorithm outlined here is named **Algo-A(ρ_n)**.

Finally, for the case of $a = 0$, no solution exists; one may want to reformulate the problem as a related lifetime maximization problem as discussed in Section II-D.

F. Finding ρ_n

The remaining step is to find ρ_n for the problem **EP**. This relies the following lemma.

Lemma 1: For each k, $1 \le k \le n$, ρ_k and $\rho_k G_k$ are monotone non-decreasing functions of ρ_n.

Proof: First, note that $G_n(\rho_n) = S_n(\rho_n) = K/\rho_n^a$ is monotone non-increasing in ρ_n for $0 \le a \le 1$. Also, since $P_n = P_n^t$ and by (6), P_n is monotone non-increasing in ρ_n. The total outgoing traffic rate per unit area in ring n is $\rho_n G_n = K\rho_n^{1-a}$, which is monotone non-decreasing in ρ_n. Since P_n is monotone non-increasing in ρ_n and $P_j = P_n$, P_j must be monotone non-increasing in ρ_n.

We make the induction hypothesis that ρ_k and $\rho_k G_k$ are both monotone non-decreasing in ρ_n for $k = j+1, \ldots, n$, where $1 \le j \le n-1$. We will show the same are true for $\rho_j G_j$ and ρ_j.

The relayed traffic rate per unit area in ring j is $\rho_j C_j$. By (3),

$$\rho_j C_j = \frac{\displaystyle\sum_{k=j+1}^{\min(n,j+l)} \rho_k (2k-1) G_k F_k(j)}{(2j-1)}. \qquad (19)$$

Hence, under the induction hypothesis, $\rho_j C_j$ is monotone non-decreasing in ρ_n.

Now, suppose we increase ρ_n to ρ_n'. Let the node densities (for achieving equal energy consumption rates) corresponding to ρ_n be denoted by $\rho_j(\rho_n)$ for $1 \le j \le n$; let the node densities corresponding to ρ_n' be denoted by $\rho_j(\rho_n')$ for $1 \le j \le n$. We note that $\rho_n(\rho_n) = \rho_n$ and $\rho_n(\rho_n') = \rho_n'$.

Then, under the induction hypothesis, $\rho_j(\rho_n) C_j(\rho_n) \le \rho_j(\rho_n') C_j(\rho_n')$. That is, under the new solution $(\rho_j(\rho_n'))_{1 \le j \le n}$, the relayed traffic rate per unit area in ring j is greater than or equal to what it is under $(\rho_j(\rho_n))_{1 \le j \le n}$. We also know that $P_j(\rho_n') = P_n(\rho_n') \le P_n(\rho_n) = P_j(\rho_n)$. This implies that $\rho_j' \ge \rho_j$. If $\rho_j' < \rho_j$ were true, then under $(\rho_j(\rho_n'))_{1 \le j \le n}$, each node in ring j would have to receive and relay more traffic per unit time, and the locally generated traffic rate $K/\rho_j(\rho_n')^a$ would also be higher, and hence, the energy consumption rate per node would be higher, i.e., $P_j(\rho_n') > P_j(\rho_n)$, which is a contradiction. We conclude that $\rho_j(\rho_n)$ is monotone non-decreasing in ρ_n.

Next, by (4) and (2),

$$\rho_j G_j = K\rho_j^{1-a} + \frac{\sum_{k=j+1}^{\min(n,j+l)} \rho_k (2k-1) G_k F_k(j)}{(2j-1)}.$$

Hence, $\rho_j G_j$ is monotone non-decreasing in ρ_n as well. ■

Hence, ρ_n can be found by binary search until the minimum node density condition (11) is satisfied. We next summarize the details of the entire algorithm.

Summary of Algorithm:

1) Initialize ρ_n to be some positive value $u > 0$ such that the problem **EP** is feasible.
   This is done as follows: Repeatedly double ρ_n; for each ρ_n, solve the corresponding ρ_j for $1 \le j < n-1$ using **Algo-A(ρ_n)** until the problem **EP** is feasible.
2) Perform binary search on $[0, u]$ for the smallest value ρ_n such that condition (11) is satisfied.
   In each step of the binary search, use **Algo-A(ρ_n)** to solve the corresponding ρ_j for $1 \le j < n-1$; check whether condition (11) is satisfied; increase or decrease ρ_n accordingly.

By Lemma 1, the solution of the above algorithm minimizes the total number of sensor nodes in the network, which is the objective of the problem **EP**, and equalizes the energy consumption rates. The overall algorithm is very simple since it consists of one-level (for $a = 1$) or two-level (for $0 < a < 1$) binary search.

G. Case of Density-Dependent Routing

When the routing $(F_k(j))$ depends on the node densities, the problem of finding the node densities becomes much harder. We will illustrate a heuristic method in Section III-A2 under a more concrete setting.

III. Important Special Cases

We now consider several important special cases in which the expressions for the node densities become simpler and more explicit. Our experiments are done on these special cases. In the following, we assume the locally generated traffic rate is $S_j(\rho_j) = K/\rho_j$ for each ring j. Furthermore, we assume the energy expenditure is dominated by transmissions, which is so when the transmission distances are reasonably large, and hence, we can set $\gamma = 0$.

A. Under Generic Routing

From (4), we can write, for all $1 \le j < n$,

$$\rho_j = \frac{K}{G_j} + \sum_{k=j+1}^{\min(n,j+l)} \frac{(2k-1) G_k F_k(j)}{(2j-1) G_j} \rho_k. \qquad (20)$$

The energy consumption rate for a node in ring j, $1 \le j \le n$, is

$$P_j = w^\alpha G_j \sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i). \qquad (21)$$

By setting $P_j = P_n$ for each j, $1 \le j < n$, we get the following relationship:

$$G_j = \frac{\displaystyle\sum_{i=(n-l)_+}^{n-1} (n-i)^\alpha F_n(i)}{\displaystyle\sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i)} G_n. \qquad (22)$$

Note that $G_n = K/\rho_n$, which depends only on ρ_n.

1) Case of Density-Independent Routing: Hence, if $F_k(j)$ has no dependency on any node density, then under a given ρ_n, one can compute G_j using (22) for all j. After that, one can compute the densities ρ_j iteratively using (20) for all j from $n-1$ down to 1.

The above assumes a fixed ρ_n. We then have to decide on ρ_n. In this special case, it turns out there is no need to do binary search for ρ_n. First, note that

$$\frac{G_k}{G_j} = \frac{\sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i)}{\sum_{i=(k-l)_+}^{k-1} (k-i)^\alpha F_k(i)}.$$

Then, (20) can be re-written as:

$$\begin{aligned}\rho_j =&\rho_n \frac{\sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i)}{\sum_{i=(n-l)_+}^{n-1} (n-i)^\alpha F_n(i)} + \\ &\sum_{k=j+1}^{\min(n,j+l)} \frac{\sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j(i)}{\sum_{i=(k-l)_+}^{k-1} (k-i)^\alpha F_k(i)} \frac{(2k-1)}{(2j-1)} F_k(j)\rho_k.\end{aligned} \tag{23}$$

It can be observed that if $(\rho_j)_{j=1}^{n-1}$ is a solution to (23) given ρ_n, then $(\kappa\rho_j)_{j=1}^{n-1}$ is a solution to (23) given $\kappa\rho_n$. One can decide the node densities as follows: Choose an arbitrary positive value for ρ_n; compute all ρ_j for $1 \le j < n$; find the minimum constant κ such that $\kappa\rho_j \ge \Theta$ for all j; use $(\kappa\rho_j)_{j=1}^{n}$ as the node densities for deployment.

2) Case of Density-Dependent Routing: The situation becomes more complicated if the routing $(F_k(j))$ depends on the node densities. Expression (23) still holds. But, it does not imply that, if ρ_n is given, then one can compute the densities ρ_j for all other j, because each $F_k(j)$ may depend on the densities in complicated ways. The set of equations in (23), for $1 \le j < n$, is a fairly complex system of nonlinear equations with ρ_j as the variables, and it is not clear how the equations can be solved.

However, (23) does suggest a different kind of iterative method to compute each ρ_j, which will be called *successive substitution*. Suppose, we initialize the iteration at some constant $\rho_j^{(0)}$ for all $1 \le j < n$. This gives $F_k^{(0)}(j)$, for different k and j. Then, one can substitute $\rho_j^{(0)}$ and $F_k^{(0)}(j)$ into the right hand side of (23) and derive $\rho_j^{(1)}$ for all $1 \le j < n$. In a general iteration step t, the following iteration occurs, for $1 \le j < n$.

$$\begin{aligned}\rho_j^{(t+1)} =&\rho_n \frac{\sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j^{(t)}(i)}{\sum_{i=(n-l)_+}^{n-1} (n-i)^\alpha F_n^{(t)}(i)} + \\ &\sum_{k=j+1}^{\min(n,j+l)} \frac{\sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha F_j^{(t)}(i)}{\sum_{i=(k-l)_+}^{k-1} (k-i)^\alpha F_k^{(t)}(i)} \frac{(2k-1)}{(2j-1)} \\ &\times F_k^{(t)}(j)\rho_k^{(t)},\end{aligned} \tag{24}$$

where, for a fixed t, $(F_k^{(t)}(j))$ is computed using $(\rho_j^{(t)})$. The process can continue until $\rho_j^{(t)}$ converges, for $1 \le j < n$. We take the values in the limit as the solutions of the equations in (23). Although we have not been able to prove convergence due to technical difficulties, we will show by experiments (Section IV) that successive substitution indeed works for all the test cases.

For the case of density-dependent routing, in general, it can be difficult to decide the final ρ_n. We consider the special case where $(F_k(j))$ is independent of the scaling of the node densities. That is, $F_k(j)$ remains unchanged for all k and j when ρ_i is scaled by a constant factor $\kappa > 0$ for all i, $1 \le i \le n$. As a result, if $(\rho_j)_{j=1}^{n-1}$ is a solution to (23) given ρ_n, then $(\kappa\rho_j)_{j=1}^{n-1}$ is a solution to (23) given $\kappa\rho_n$. The method to decide the final node densities is the same as the case of density-independent routing; i.e., we choose an arbitrary positive value for ρ_n, compute all ρ_j for $1 \le j < n$, and find the minimum constant κ such that $\kappa\rho_j \ge \Theta$ for all j.

If $(F_k(j))$ is dependent of the scaling of the node densities, then, an ad-hoc method is to try different values of ρ_n and conduct the successive substitution procedure for each ρ_n until a satisfactory set of ρ_j is found.

B. Several Routing Schemes

Expression (23) is for a generic routing $(F_k(j))$. For any routing scheme used in practice, if one can cast it into a specification in terms of $(F_k(j))$, then one can use (5), (22) and (20) (or, equivalently, (23)) to compute the required node densities, as outlined in Section III-A. Next, we will consider some simple routing schemes as examples and later show numerical results about them. Many more routing schemes can be considered similarly.

1) Uniform Ring Selection: With this scheme, a node finds its next-hop node in the direction to the sink via two steps. First, the node selects a reachable ring with a uniform probability distribution. Second, the sending node randomly chooses the next-hop node among the nodes in the intersection of the selected ring and the sender's communication range. The probability that ring i, $(j-l)_+ \le i < j$, is selected as the next-hop ring by a node in ring j is

$$F_j(i) = 1/\min(l, j).$$

Note that, in this case, $F_j(i)$ is independent on the node densities.

From (23), the node densities can be computed iteratively from $n-1$ to 1 by the following expression.

$$\begin{aligned}\rho_j =&\rho_n \frac{\min(n,l)}{\min(j,l)} \frac{\sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha}{\sum_{i=(n-l)_+}^{n-1} (n-i)^\alpha} + \\ &\sum_{k=j+1}^{\min(n,j+l)} \frac{\sum_{i=(j-l)_+}^{j-1} (j-i)^\alpha}{\sum_{i=(k-l)_+}^{k-1} (k-i)^\alpha} \frac{(2k-1)}{(2j-1)} \frac{1}{\min(j,l)} \rho_k.\end{aligned}$$

2) Uniform Node Selection: In this routing scheme, a node X can select any node with the same probability as long as the target node resides in X's communication range and is closer to the sink than X is. This scheme is motivated by geographical routing.

Figure 2 illustrates the underlying geometry for Uniform Node Selection. Node X can choose any node in the shaded

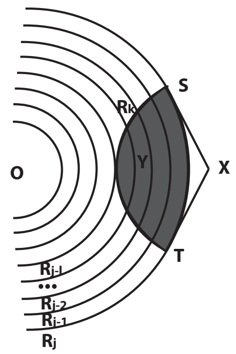


Fig. 2. Uniform Node Selection

part as the next-hop node. Denote by Q the region within the communication range of node X. The shaded region is the intersection of Q and the inner rings that X can reach, i.e, $\bigcup_{k=(j-l)_+}^{j-1}(R_k \cap Q)$. The number of nodes in $R_k \cap Q$ is $\rho_k A(R_k \cap Q)$, where the notation $A(U)$ represents the area of a region U. The number of possible next-hop nodes for node X is $\sum_{k=(j-l)_+}^{j-1} \rho_k A(R_k \cap Q)$. Hence, the probability $F_j(i)$ is, for $(j-l)_+ \leq i < j$,

$$F_j(i) = \frac{\rho_i A(R_i \cap Q)}{\sum_{k=(j-l)_+}^{j-1} \rho_k A(R_k \cap Q)}. \tag{25}$$

From basic knowledge of geometry, we can find the area of the intersection of two disks [12]. If the distance between the centers of two disks of radii r and R, respectively, is d, the area of the intersection is given by

$$\begin{aligned} &\theta(r, R, d) \\ &= r^2 \cos^{-1}\left(\frac{d^2 + r^2 - R^2}{2dr}\right) + R^2 \cos^{-1}\left(\frac{d^2 + R^2 - r^2}{2dR}\right) \\ &\quad - \frac{1}{2}\sqrt{(-d + r + R)(d + r - R)(d - r + R)(d + r + R)}. \end{aligned}$$

Now, we can find the area of $R_k \cap Q$. Let d be the distance of node X from the sink and note that lw is the communication range of node X in units of distance. The area of $R_k \cap Q$, for $(j-l)_+ < k < j$, is obtained by,

$$A(R_k \cap Q) = \theta(lw, kw, d) - \theta(lw, (k-1)w, d). \tag{26}$$

The area of $R_{(j-l)_+} \cap Q$ is given by

$$A(R_{(j-l)_+} \cap Q) = \theta(lw, w(j-l)_+, d). \tag{27}$$

The probability $F_j(i)$ can be obtained by plugging expressions (26) and (27) into (25). By applying the expressions for $F_j(i)$ to (23), we derive a set of equations in ρ_j only, for different j. From (25), note that $F_j(i)$ depends on various ρ_i. In our experiment, we solved the equations by successive substitution, which is to iterate $(\rho_j^{(t)})$ over t as in (24).

IV. Experimental Results

In this section, we use simulation and numerical experiments to verify whether our method for computing the node densities works. The radius of the sensor field is 50 (units) and the total number of rings is 20. The procedure of the experiments is as follows. First, for a given routing scheme and experimental parameters, we calculate the node densities of the rings using the equations and method introduced in Sections II and III. Then, in our simulation setup, we randomly deploy the sensor nodes into the sensor field according to the calculated densities and have each node select its next-hop neighbor according to the given routing scheme. The density of the outmost ring ρ_n can be tuned to control the total number of nodes in the sensor field. In the simulation run, we measure the energy consumption rate of each node. Finally, we compute the average per-node energy consumption rate for each ring. The goal is to verify whether the calculated densities result in an even energy consumption rate in all rings.

A. Uniform Ring Selection

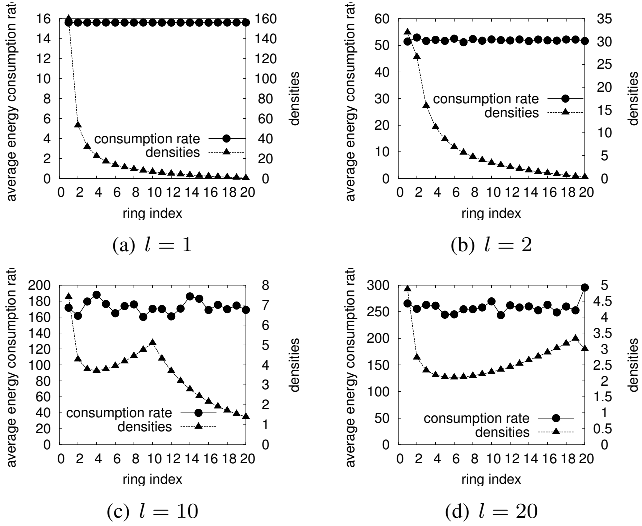
The results for various maximum ranges l are shown in Figure 3, where the path loss exponent, α, is 2. We have conducted extensive experiments for other values of α. The results for the case of $\alpha = 2$ are representative and, for brevity, we omit the results for other values of α. In Figure 3, we show both the average per-node energy consumption rate and the calculated node density in each of the rings. Several observations can be made. First, the average per-node energy consumption rates of the rings are nearly identical. This demonstrates that our modeling approach and analytical method are highly accurate, and that correct node densities can be derived from the resulting mathematical expressions or methods. Second, the shape of the density function, as a function of the ring index, is somewhat surprising in some cases. The functions are not even monotonic in the case of $l = 10$ or $l = 20$.

In the cases of $l = 1$ or $l = 2$, the density function is monotonic and increases very fast as the ring gets closer to the sink. It is easy to explain the case of $l = 1$. Since the maximum range is 1, all the traffic of a node must flow through the adjacent ring on the inside. Therefore, the traffic load becomes heavier as the ring gets closer to the sink. It is necessary to deploy more nodes in the rings closer to the sink so as to balance the energy consumption rates across the rings. As it approaches the sink, the area of the ring decreases while the number of nodes in the ring increases. Hence, the density increases fast.

For larger values of l, e.g., $l = 10$, it is not necessarily true that higher node densities are required for rings closer to the sink. This is more due to the "boundary effect". In such a case, each node can directly transmit its traffic to multiple inside rings. However, longer transmission distance requires more energy. A node in one of the l inner-most rings (R_i, $1 \leq i \leq l$) has fewer than l rings left on the inside. Hence, its maximum transmission distance is actually less than l rings away. As a result, it tends to consume less energy on average than a node in a ring further outside, say R_j for $j > l$. The precise situation is complicated, depending on the parameters of the energy consumption model and the routing probabilities.

B. Uniform Node Selection

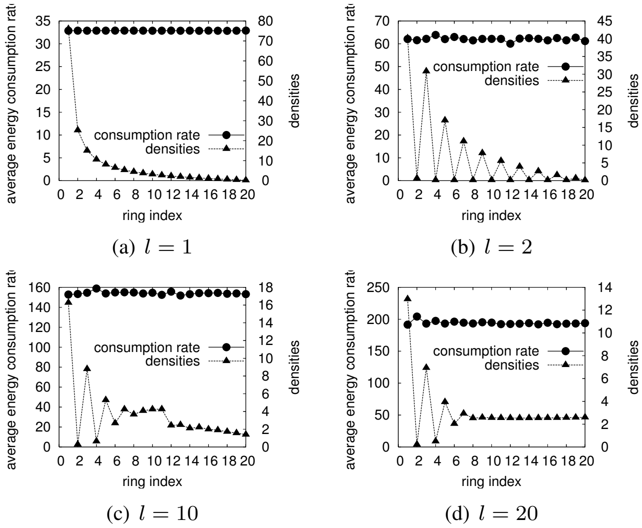
Unlike the case of Uniform Ring Selection, here, the node densities are computed by successive substitution as in (24).



(a) $l = 1$ (b) $l = 2$ (c) $l = 10$ (d) $l = 20$

Fig. 3. Node densities and average per-node energy consumption rates for various maximum ranges, l, under Uniform Ring Selection. $\alpha = 2$.

The results for $\alpha = 2$ are shown in Figure 4. In all plots, the curve for the average per-node energy consumption rate is flat. This means that, if we deploy the nodes according to the computed densities, we can achieve an even energy consumption rate in all rings. The results indicate that our modeling approach, analytical method and numerical solutions are all sound. Note that the curves for the node densities can be quite oscillatory or irregular. Hence, it is quite hard to predict the deployment densities using ad hoc approaches. Our method for precise computation can be valuable.



(a) $l = 1$ (b) $l = 2$ (c) $l = 10$ (d) $l = 20$

Fig. 4. Node densities and average per-node energy consumption rates for various maximum ranges, l, under Uniform Node Selection. $\alpha = 2$.

V. Conclusions

In this paper, we examine how to apply non-uniform deployment of the sensor nodes to resolve the problem of uneven energy consumption rates by the nodes, or the energy hole problem, in multi-hop wireless sensor networks. More generally, non-uniform deployment with careful density control can be an important technique for achieving a desirable lifetime and system-cost tradeoff for a sensor network. Our main contribution is to present a method for computing the required node density function. As an example, we show that the method enables us to compute the correct densities that achieve an equal energy consumption rate for all nodes, thereby, extending the network lifetime. The method is expected to be applicable to other similar objectives.

We next discuss some limitations of the paper. The analytical approach is essentially of a mean-value type and it is expected to be accurate for networks with a large number of densely deployed sensors. One limitation is that it ignores statistical fluctuations, which may be an important factor of consideration for low-density networks. Nevertheless, our experimental results have shown the effectiveness of the approach for networks of moderate density. For tractability, the method requires a model of the underlying routing protocol in the form of the probabilistic model given in the paper. The probabilistic model is meant to be flexible enough for capturing the essence of many practical routing protocols. However, it may be not powerful enough to capture sufficient details of some routing protocols. Some other issues are also overlooked in this paper, such as the algorithms and protocols for determining which nodes become active or inactive in each region. These issues are either orthogonal to this work or left for future studies. Finally, the ring-type network model is a limitation and our ongoing work is addressing that.

References

[1] J. H. Chang and L. Tassiulas, "Maximum lifetime routing in wireless sensor networks," *IEEE/ACM Transactions of Networking*, vol. 12, pp. 609–619, August 2004.

[2] ——, "Routing for maximum system lifetime in wireless ad hoc networks," in *37th Annual Allerton Conference on Communication, Control, and Computing*, September 1999.

[3] A. Giridhar and P. R. Kumar, "Maximizing the functional lifetime of sensor networks," in *The 4th International Symposium on Information Processing in Sensor Networks, 2005*, April 2005, pp. 5–12.

[4] W. R. Heinzelman, A. Chadrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for wireless microsensor networks," in *Proceedings of the 33rd Hawaii International Conference on System Sciences*, January 2000.

[5] A. Sankar and Z. Liu, "Maximum lifetime routing in wireless ad-hoc networks," in *Proceedings of IEEE INFOCOM*, 2004.

[6] Y. Xue, Y. Cui, and K. Nahrstedt, "Maximizing lifetime for data aggregation in wireless sensor networks," *Mobile Networks and Applications*, vol. 10, no. 6, pp. 853–864, December 2005.

[7] L. Popa, A. Rostamizadeh, R. M. Karp, and C. Papadimitriou, "Balancing traffic load in wireless networks with curveball routing," in *Proceedings of ACM MobiHoc*, Sept 2007, pp. 170–179.

[8] S. Olariu and I. Stojmenovic, "Design guidelines for maximizing lifetime and avoiding energy holes in sensor networks with uniform distribution and uniform reporting," in *Proceedings of IEEE INFOCOM*, May 2006.

[9] J. Li and P. Mohapatra, "Analytical modeling and mitigation techniques for the energy hole problem in sensor networks," *Pervasive and Mobile Computing*, vol. 3, no. 8, pp. 233–254, 2007.

[10] X. Wu, G. Chen, and S. K. Das, "Avoiding energy holes in wireless sensor networks with nonuniform node distribution," *IEEE Transactions on Parallel and Distributed Systems*, vol. 19, no. 5, pp. 710–720, May 2008.

[11] J. Lian, K. Naik, and G. B. Agnew, "Data capacity improvement of wireless sensor networks using non-uniform sensor distribution," *International Journal of Distributed Sensor Networks*, vol. 2, April-June 2006.

[12] "Wolfram mathworld: Circle-circle intersection," http://mathworld.wolfram.com/Circle-CircleIntersection.html.