GUC-Secure Commitments via Random Oracles: New Impossibility and Feasibility*

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Abstract

In the UC framework, protocols must be subroutine respecting; therefore, shared trusted setup might cause security issues. To address this drawback, Generalized UC (GUC) framework is introduced by Canetti *et al.* (TCC 2007). In this work, we investigate the impossibility and feasibility of GUC-secure commitments using global random oracles (GRO) as the trusted setup. In particular, we show that it is impossible to have a 2-round (1-round committing and 1-round opening) GUC-secure commitment in the global observable RO model by Canetti *et al.* (CCS 2014). We then give a new round-optimal GUC-secure commitment that uses only Minicrypt assumptions (i.e. the existence of one-way functions) in the global observable RO model. Furthermore, we also examine the complete picture on round complexity of the GUC-secure commitments in various global RO models.

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Contents

1 Introduction

Secure multi-party computation (MPC) [Yao82, GMW87] is one of the most important cornerstone of modern cryptography. It enables n mutually distrustful players, $P_1, \ldots, P_n$ to securely evaluate any efficiently computable function f of their private inputs, $x_1, \ldots, x_n$. Since its introduction in the early 1980s, MPC has been extensively studied in the literature. Typically, the security properties of an MPC protocol are formalized using the well-known "simulation-paradigm" [GMR89, GMW87]. Roughly speaking, the idea is to require that any adversarial attacker $\mathcal{A}$ in the *real world* execution of the protocol, can be emulated by a so-called "simulator" $\mathcal{S}$ in an *ideal world* execution, where the players provide their inputs to a trusted third party who computes f for them and relays the result back to the players.

From UC to GUC. To facilitate modular protocol design and analysis in the complex network environments, Canetti proposed the Universal Composibility (UC) framework [Can01], where, the notion of indistinguishability between the real and the ideal world is replaced by a notion of "interactive indistinguishability". More specifically, an interactive *environment*, which may communicate with both the honest players and the corrupted ones, should not be able to distinguish whether it is participating in the real execution or the ideal one. UC security guarantees the security of the MPC protocols under *concurrent executions*, and even other *arbitrary* protocols running in the same network cannot be adversarially affected — roughly speaking, the environment represents the collection of any other concurrent protocols. Additionally, this notion is closed under composition, enabling modular analysis of protocols.

However, protocols in the UC framework must be *subroutine respecting*, and shared setup cannot be directly modeled by the basic UC notion. To address this drawback, Canetti, Dodis, Pass and Walfish proposed the Generalized Universal Composibility (GUC) framework in 2007 [CDPW07]. Since then, many interesting and efficient protocols have been designed and analyzed under the GUC framework [DSW08, CJS14, MRS17, CDG+18, CSW20].

Random Oracles as a global setup: $\mathcal{G}_{\mathsf{sRO}}$, $\mathcal{G}_{\mathsf{oRO}}$, $\mathcal{G}_{\mathsf{pRO}}$, and $\mathcal{G}_{\mathsf{poRO}}$. It has been shown [CF01, CDPW07] that, to achieve secure multi-party computation for any non-trivial functionality in the UC and the GUC framework, certain trusted setups (e.g., CRS, PKI, etc.) are required. Random Oracle (RO) is a classic idealized setup that can be used to design UC-secure [HM04] and GUC-secure multi-party computation protocols [CJS14, CDG+18].

Random oracle model [BR93] is a popular idealized model that has been widely used to justify the security of efficient cryptographic protocols. In spite of its known inability to provide provable guarantees when RO is instantiated with a real-world hash function [CGH98], RO is still a promising setup without known real-world attacks. In fact, RO draws increasing attention along with recent advancement of the blockchain technology. It is generally viewed as a *transparent* setup that can be easily deployed with no reliance on any trusted party in the blockchain and other distributed system setting. Many RO-based non-interactive ZK systems, e.g., zk-STARK [BBHR19] and Fractal [COS20], are developed and deployed in real application scenarios. Note that, those RO-based protocols can achieve *post-quantum* security.

A natural formulation of a global RO, denoted as $\mathcal{G}_{\mathsf{sRO}}$, has been defined in [CDPW07]: it is accessible to all parties both in the ideal world and the real world, but it offers neither "observability" nor "programmability". We emphasize that, it has been proven that it is impossible to achieve GUC-secure commitment in the $\mathcal{G}_{\mathsf{sRO}}$ model [CDPW07]. Later, Canetti, Jain, and Scafuro [CJS14] proposed a strengthened version of the global RO, denoted as $\mathcal{G}_{\mathsf{oRO}}$, which allows the simulator to "observe" the queries made by the malicious parties, and GUC-secure commitment *can* be constructed in the $\mathcal{G}_{\mathsf{oRO}}$ model. Camenisch *et al.* [CDG+18] further strengthened the $\mathcal{G}_{\mathsf{sRO}}$ from

a different direction: they designed a mechanism that allows the simulator to "program" the global RO without being detected by the adversary, and we denote this strengthened version of the global RO as $\mathcal{G}_{\mathsf{pRO}}$. On top of both $\mathcal{G}_{\mathsf{oRO}}$ and $\mathcal{G}_{\mathsf{pRO}}$, Camensich *et al.* [CDG+18] then introduced an even stronger variant, called $\mathcal{G}_{\mathsf{poRO}}$, and they constructed a round-optimal GUC-secure commitment in the $\mathcal{G}_{\mathsf{poRO}}$ model [CDG+18]. Figure 3 depicts the relation of these global RO models.

Problem statement. We study the round complexity of GUC-secure commitment in the global RO models. Clearly protocols relying on a *less idealized* setup and *weaker* computational assumptions will allow us to gain better confidence in the proved security statement. Note that, round-optimal GUC secure commitments can be constructed based on the strong global RO setup $\mathcal{G}_{\mathsf{poRO}}$ [CDG+18]. On the other hand, in [CDPW07], it has been proven that constructing a GUC-secure commitment in the $\mathcal{G}_{\mathsf{sRO}}$ model is impossible. Between these two extremes, in [CJS14], Cannetti et al have shown that it is feasible to construct a GUC-secure commitment in the $\mathcal{G}_{\mathsf{oRO}}$ model; however, their construction relies on the discrete logarithm assumption, which cannot achieve (post-) quantum security. We are interested in GUC-secure commitment protocols using a global RO setup and Minicrypt [Imp95] assumptions; these protocols can additionally achieve post-quantum security. This leads us to a natural research question:

> *What is the lower bounds of the round complexity*[1] *of a GUC-secure commitment in the* $\mathcal{G}_{\mathsf{oRO}}$ *model?*

If there exists such a lower bound on the round complexity of a GUC-secure commitment in the $\mathcal{G}_{\mathsf{oRO}}$ model, we would like to find a round-optimal construction. We hereby ask:

> *If there exists such a lower bound, is that possible to construct round-optimal GUC-secure commitment in the* $\mathcal{G}_{\mathsf{oRO}}$ *model, using only Minicrypt assumption?*

1.1 Our Results

We give affirmative answers to the above research questions. Our findings can be summarized as follows.

A new impossibility result in the $\mathcal{G}_{\mathsf{oRO}}$ model. In this work, we show that 2-round (1-round for committing and 1-round for opening) GUC-secure commitment does not exist in the $\mathcal{G}_{\mathsf{oRO}}$ model (cf. Section 3).

We prove this result by contradiction, and our main observation is as follows. Suppose such a 2-round GUC-secure commitment exists. First, it is easy to see that if the committing phase only takes one round, then there is only one message sent from the committer to the receiver; that is, the receiver does not send any message to the committer. Analogously, the receiver is also "silent" in the 1-round opening phase. Therefore, the potentially corrupted receiver can delay all its $\mathcal{G}_{\mathsf{oRO}}$ queries until it receives the opening message from the committer.

Let us consider the case where the receiver is corrupted. During the simulation, the simulated committer needs to generate the commitment message without the knowledge of the plaintext, and it later needs to generate the opening message for any given input (a.k.a. the plaintext). As discussed before, the corrupted receiver can choose not to query the $\mathcal{G}_{\mathsf{oRO}}$ until the simulator has equivocated the commitment. Hence, the simulator cannot obtain any illegitimate queries from $\mathcal{G}_{\mathsf{oRO}}$ for this corrupted receiver to facilitate this equivocation. Now, observe that this simulator

[1] Throughout this work, we do not consider the case of simultaneous rounds where two parties can send their messages to each other at the same round [GIS18, MR19].

has no extra power over a normal party; in particular, any committer can invoke such a simulator (algorithm) to violate the binding property of the commitment.

In the actual proof of our impossibility result, we let the corrupted committer to internally run the simulator algorithm to generate the commitment message, providing an empty list for the $\mathcal{G}_{\mathsf{oRO}}$ illegitimate queries. Obviously, given this commitment message, the receiver/simulator cannot extract its plaintext; Therefore, with very high probability, the simulation would fail.

A new round-optimal commitment using $\mathcal{G}_{\mathsf{oRO}}$. With respect to our impossibility result, a round-optimal commitment should takes at least 3 rounds. In this work, we show how to construct a round-optimal (2-round for committing and 1-round for opening) GUC-secure commitment only using Minicrypt assumptions in the $\mathcal{G}_{\mathsf{oRO}}$ model (cf. Section 4).

A general framework. A typical GUC-secure commitment requires both extractability and equivocality. The $\mathcal{G}_{\mathsf{oRO}}$ model can directly provide the simulator with extractability; therefore, the challenge is to design an equivocation mechanism with round efficiency. A natural approach is to utilize a (property-based) perfect hiding non-interactive equivocal commitment: (i) in the 1st round, the receiver picks the commitment key and sends it to the committer; and (ii) in the 2nd round, the committer uses the equivocal commitment scheme to commit the message. To deploy this approach, the following questions need to be resolved:

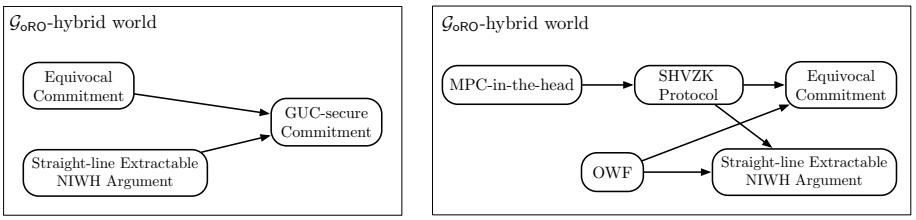
- *How to instantiate such a perfect-hiding non-interactive equivocal commitment?*
- *How can the simulator obtain the equivocation trapdoor?*

In [CJS14] and [MRS17], the Pedersen commitment is used as a candidate of the equivocal commitment. It is well-known, the security of Pedersen commitment is based on the discrete logarithm assumption which is not (post-) quantum secure. In this work, we show how to construct a candidate of the equivocal commitment only using Minicrypt assumptions, i.e. the existence of one-way functions, in the $\mathcal{G}_{\mathsf{oRO}}$ model.

To address the latter question, [CJS14] introduced a 5-round mechanism that enables the simulator to obtain the equivocation trapdoor in the $\mathcal{G}_{\mathsf{oRO}}$ model; whereas, [MRS17] proposed a more round-efficient (3-round) mechanism to do so. More precisely, [MRS17] let the receiver use a Non-Interactive Witness Indistinguishable (NIWI) argument to prove the knowledge of equivocation trapdoor w.r.t. the commitment key. The proof is sent together with the commitment key in the 1st round. Note that straight-line extractability is needed for this approach.

Following the technique proposed in [MRS17], our framework adopts the Non-Interactive Witness Hiding (NIWH) argument with straight-line extractability [Pas03] to prove the knowledge of equivocation trapdoor w.r.t. the commitment key. The straight-line extractable NIWH argument can be constructed under Minicrypt assumption in the $\mathcal{G}_{\mathsf{oRO}}$ model. Putting things together, we can obtain a GUC-secure commitment using only Minicrypt assumptions. We present the technique roadmap of our framework in Figure 1.

Non-interactive equivocal commitment in Minicrypt. As shown in [Dam02, MY04], it is possible to build a non-interactive equivocal commitment from a 3-round public-coin Special Honest Verifier Zero-Knowledge (SHVZK) protocol with 2-special soundness. In the SHVZK protocol, the prover sends the message flow a in the 1st round, and the receiver sends a public-coin randomness e as the challenge in the 2nd round. After receiving e, the prover computes and sends the response z in the last round. The technique of constructing non-interactive equivocal commitment can be



(a) A general framework. (b) Instantiation of building blocks.

Figure 1: Technique Roadmap

summarized as follows. Let $\mathcal{R}_{\mathcal{L}}$ be an NP relation whose associate language is $\mathcal{L}$. The receiver randomly samples a pair $(x, w) \in \mathcal{R}_{\mathcal{L}}$ and sends x to the committer. To commit a message m, the committer invokes the SHVZK simulator for $x \in \mathcal{L}$, using m as the challenge. The simulator then outputs the simulated proof (a, z). The committer sends a to the receiver as its commitment message. To open the commitment, the committer can simply send m, z to the receiver, who will accept it if and only if (a, m, z) is an accepting SHVZK proof transcript. The equivocation trapdoor is w, which can be extracted from the straight-line extractor of NIWH as described above.

Since we aim to construct a commitment under Minicrypt assumptions, in our construction, $\mathcal{R}_{\mathcal{L}}$ is instantiated with a one-way function relation, i.e., $g(x) = y$ where g is a one-way function. Next, how to construct a 2-special sound SHVZK protocol under Minicrypt assumptions? One possible approach is to use the "MPC-in-the-head" paradigm proposed by Ishai *et al.* [IKOS07]. Roughly speaking, the main idea is for the prover to simulate the execution of an n-party computation protocol that checks if $(x, w) \in \mathcal{R}_{\mathcal{L}}$, where x is the public input and w is the witness. The prover then commits to all views of the parties and sends the commitments to the verifier. After that, the verifier chooses a random subset of the parties and asks the prover to open their corresponding views. The verifier accepts the proof if the revealed views are consistent. Unfortunately, to the best of our knowledge, none of the followups [GMO16, CDG+17, AHIV17, KKW18, dOT21] since the initial work of [IKOS07] can lead to a 2-special sound SHVZK protocol merely under Minicrypt assumptions. To address this issue, we propose a new technique that can construct a 2-special sound protocol in the $\mathcal{G}_{\mathsf{oRO}}$ model (cf. Section 4.2.1).

Towards a complete picture. In terms of the $\mathcal{G}_{\mathsf{oRO}}$, our work gives a complete answer to our questions: we show there exists *no* 2-round GUC-secure commitment in the $\mathcal{G}_{\mathsf{oRO}}$ model (cf. Section 3), and present a 3-round (round-optimal) GUC-secure commitment under only Minicrypt assumptions in the $\mathcal{G}_{\mathsf{oRO}}$ model (cf. Section 4). Moreover, it is known that GUC-secure commitment does not exist in the $\mathcal{G}_{\mathsf{sRO}}$ model [CDPW07], and round-optimal GUC-secure commitment can be constructed without further assumptions in the $\mathcal{G}_{\mathsf{poRO}}$ model [CDG+18]. What about the $\mathcal{G}_{\mathsf{pRO}}$? In this work, we also show some impossibility result: there exists *no* GUC-secure commitment with 1-round committing in the $\mathcal{G}_{\mathsf{pRO}}$ model (see details in the full version of our paper). However, the feasibility of round-optimal GUC-secure commitment under Minicrypt assumptions in the $\mathcal{G}_{\mathsf{pRO}}$ model remains an open question.

Further investigation and future directions. We mainly focus on the commitment in this work. One may also wonder the lower bounds of the round complexity of other cryptographic primitives such as ZK, OT, etc. In fact, it is already known that there exists no NIZK in the observable RO model [Pas03]. What about the ZK proofs in the $\mathcal{G}_{\mathsf{pRO}}$ model? In this work, we show that our impossibility result can be extended to ZK proofs in the $\mathcal{G}_{\mathsf{pRO}}$ model: there exists

no non-trivial GUC-secure NIZK protocols in the $\mathcal{G}_{\mathsf{pRO}}$ model (see details in the full version of our paper).

1.2 Related Work

In terms of UC security with local setups, non-interactive commitments (1-round for committing and 1-round for opening) can be constructed under various setup assumptions. For instance, Canetti and Fischlin gave a candidate in the CRS model [CF01]; Hofheinz and Müller-Quade suggested a candidate in the RO model [HM04]. As for UC security with global setups, it is still unclear if it is possible to construct a non-interactive GUC-secure commitment, and very few work, e.g., [DSW08] is dedicated to this research area. In [CDPW07], Canetti *et al.* showed that it is impossible to construct a GUC-secure commitment merely relying on local CRS/RO functionalities; they further proposed a 7-round GUC-secure commitment protocol in the Agumented CRS (ACRS) model. Later, Dodis *et al.* proved that there exists *no* GUC-secure commitment with 1-round committing phase in the ACRS model against adaptive adversaries [DSW08]. Note that their impossibility result can be extended to any other global setup whose output depends on the program ID (pid) of the querying party, but not the session ID (sid), such as the Key Registration of Knowledge (KRK) model [BCNP04]. To bypass this impossibility result, $\mathcal{G}_{\mathsf{oRO}}$,$\mathcal{G}_{\mathsf{pRO}}$ and $\mathcal{G}_{\mathsf{poRO}}$ are proposed; note that the output of those setup functionalities depends on the session ID (sid).

Focusing on commitments in the $\mathcal{G}_{\mathsf{oRO}}$, Canetti *et al.* proposed a 5-round GUC-secure commitment [CJS14]. Later, Mohassel *et al.* gave a $(1+2)$-round GUC-secure commitment in the $\mathcal{G}_{\mathsf{oRO}}$ model, where the committer and the receiver needs to have an additional 1-round setup phase followed by a 2-round commitment [MRS17]. Note that their construction also employed Pedersen commitment, which cannot achieve (post-) quantum security. Byali *et al.* gave a 2-round GUC-secure commitment construction in the CRS and $\mathcal{G}_{\mathsf{oRO}}$ hybrid model [BPRS17]. Following Byali *et al.* paradigm, GUC-secure ZK protocols [GKPS18, LR22] can also be constructed in the CRS and $\mathcal{G}_{\mathsf{oRO}}$ hybrid model. With regard to post-quantum security, [BGM19] gave a 5-round lattice-based GUC-secure commitment and [Bra21] gave a 6-round code-based GUC-secure commitment in the $\mathcal{G}_{\mathsf{oRO}}$ model.

In respect of the $\mathcal{G}_{\mathsf{pRO}}$ and the $\mathcal{G}_{\mathsf{poRO}}$, Camenisch *et al.* proposed a 3-round GUC-secure commitment from CDH assumption in the $\mathcal{G}_{\mathsf{pRO}}$ model and an information-theoretical non-interactive GUC-secure commitment in the $\mathcal{G}_{\mathsf{poRO}}$ model [CDG+18]. Recently, Canetti *et al.* proposed a 2-round OT adaptive-secure OT from DDH assumption in the $\mathcal{G}_{\mathsf{pRO}}$ model [CSW20], but their protocol is only UC-secure. Baum *et al.* constructed a GUC-secure commitment scheme that is additively homomorphic in the $\mathcal{G}_{\mathsf{poRO}}$ model [BDD20].

2 Preliminaries

2.1 Notations

Let $\lambda \in \mathbb{N}$ be the security parameter. We say that a function $\mathsf{negl} : \mathbb{N} \to \mathbb{N}$ is negligible if for every positive polynomial $p(\cdot)$ and all sufficiently large λ, it holds that $\mathsf{negl}(\lambda) < \frac{1}{p(\lambda)}$. We write $y := \mathsf{Alg}(x; r)$ when the algorithm Alg on input x and randomness r, outputs y. We write $y \leftarrow \mathsf{Alg}(x)$ for the process of sampling the randomness r and setting $y := \mathsf{Alg}(x; r)$. We also write $y \leftarrow S$ for sampling y uniformly at random from the set S. We use the abbreviation PPT to denote probabilistic polynomial-time. Let $[n]$ denote the set $\{1, 2, \ldots, n\}$ for some $n \in \mathbb{N}$. For an NP relation $\mathcal{R}$, we denote by $\mathcal{L}$ its associate language, i.e. $\mathcal{L} = \{x \mid \exists w \text{ s.t. } (x, w) \in \mathcal{R}\}$. We often write $\mathcal{R}_{\mathcal{L}}$ to denote the NP relation whose associate language is $\mathcal{L}$ for short. We also use $\mathcal{R}_{\mathcal{L}}(x, w) = 1$

to refer to $(x, w) \in \mathcal{R}_{\mathcal{L}}$. We say that two distribution ensembles $\mathcal{X} = \{\mathcal{X}_\lambda\}_{\lambda\in\mathbb{N}}$ and $\mathcal{Y} = \{\mathcal{Y}_\lambda\}_{\lambda\in\mathbb{N}}$ are identical (resp. computationally indistinguishable), denoted by $\mathcal{X} \equiv \mathcal{Y}$ (resp., $\mathcal{X} \overset{c}{\approx} \mathcal{Y}$), if for any unbounded (resp., PPT) distinguisher $\mathcal{D}$ there exists a negligible function $\mathsf{negl}(\cdot)$ such that $|\Pr[\mathcal{D}(\mathcal{X}_\lambda) = 1] - \Pr[\mathcal{D}(\mathcal{Y}_\lambda) = 1]| = 0$ (resp., $\mathsf{negl}(\lambda)$). When we define a protocol/scheme in form of $\Pi = \Pi.\{\mathsf{Alg\text{-}1}, \ldots, \mathsf{Alg\text{-}n}\}$, we use the notation $\Pi.\mathsf{Alg\text{-}i}$ to refer to the algorithm $\mathsf{Alg\text{-}i}$ of Π where $\mathsf{Alg\text{-}i} \in \{\mathsf{Alg\text{-}1}, \ldots, \mathsf{Alg\text{-}n}\}$.

2.2 Universal Composability

Canetti's UC framework. The UC framework proposed by Canetti [Can01] lays down a solid foundation for designing and analyzing protocols secure against attacks in an arbitrary network execution environment. Roughly speaking, in the UC framework, a protocol Π is defined to be a computer program (or several programs) which is intended to be executed by multiple interconnected parties. Every party is identified by the unique pair $(\mathsf{pid}, \mathsf{sid})$, where pid is the Program ID (PID) and sid is the Session ID (SID). Let $\mathcal{A}$ be the adversary who can control the network and corrupt the parties. When a party is corrupted, the adversary $\mathcal{A}$ receives its private input and its internal state. We say a protocol is *terminating* if it can terminate in polynomial time, and we only consider terminating protocols in this work.

We call a protocol, the one for which we want to prove security, the challenge protocol. A challenge protocol Π is a UC-secure realization of a functionality $\mathcal{F}$, if it satisfies that for every PPT adversary $\mathcal{A}$ attacking an execution of Π, there is another PPT adversary $\mathcal{S}$—known as the simulator—attacking the ideal process that interacts with $\mathcal{F}$ (by corrupting the same set of parties), such that the executions of Π with $\mathcal{A}$ and that of $\mathcal{F}$ with $\mathcal{S}$ makes no difference to any PPT network execution environment $\mathcal{Z}$.

The ideal world execution. In the ideal world, the set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ only communicate with an ideal functionality $\mathcal{F}$ and the simulator $\mathcal{S}$. The corrupted parties are controlled by the simulator $\mathcal{S}$. The output of the environment $\mathcal{Z}$ in this execution is denoted by $\mathsf{EXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

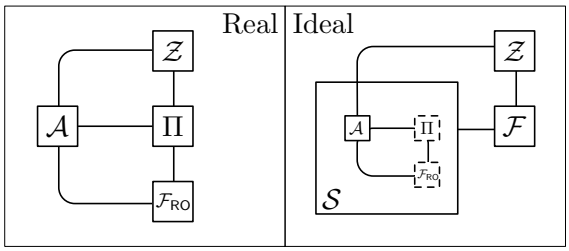
The real world execution. In the real world, the set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ communicate with each other and the adversary $\mathcal{A}$ to run the protocol Π. The corrupted parties are controlled by the adversary $\mathcal{A}$. The output of the environment $\mathcal{Z}$ in this execution is denoted by $\mathsf{EXEC}_{\Pi,\mathcal{A},\mathcal{Z}}$.

Definition 1. *We say a protocol Π UC-realizes functionality $\mathcal{F}$, if for any PPT environment $\mathcal{Z}$ and any PPT adversary $\mathcal{A}$ there exists a PPT simulator $\mathcal{S}$ s.t.* $\mathsf{EXEC}_{\Pi,\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

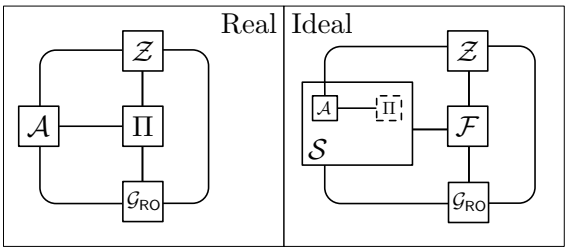
In order to conceptually modularize the design of the protocols, the notion of "hybrid world" is introduced. A protocol Π is said to be realized "in the $\mathcal{G}$ hybrid world" if Π invokes the ideal functionality $\mathcal{G}$ as a subroutine.

Definition 2. *We say protocol Π UC-realizes functionality $\mathcal{F}$ in the $\mathcal{G}$ hybrid world, if for any PPT environment $\mathcal{Z}$ and any PPT adversary $\mathcal{A}$ there exists a PPT simulator $\mathcal{S}$ s.t.* $\mathsf{EXEC}^{\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

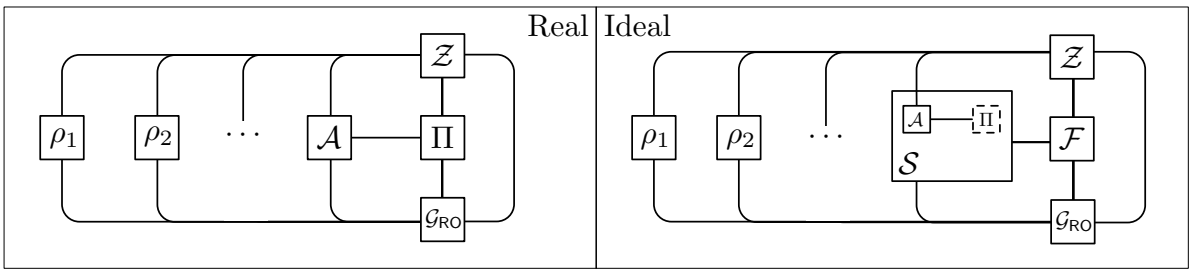
Furthermore, in the UC framework, the environment $\mathcal{Z}$ cannot have the direct access to $\mathcal{G}$, but it can do so through the adversary. Namely, in the real world, the adversary $\mathcal{A}$ can access the ideal functionality $\mathcal{G}$ directly, and $\mathcal{A}$ queries $\mathcal{G}$ for $\mathcal{Z}$ and forwards the answers; analogously, in the ideal world, $\mathcal{Z}$ can query $\mathcal{G}$ through the simulator $\mathcal{S}$. This implicitly means that $\mathcal{G}$ is local to the challenge protocol instance. This allows the simulator $\mathcal{S}$ to simulate $\mathcal{G}$ in the ideal world as long as it "looks" indistinguishable from $\mathcal{G}$ hybrid world.



(a) Basic UC: the simulator $\mathcal{S}$ simulates the local $\mathcal{F}_{\mathsf{RO}}$ and has full control.



(b) EUC: the global $\mathcal{G}_{\mathsf{RO}}$ is external to the simulator, and the environment $\mathcal{Z}$ is $\mathcal{G}$-externalized constrained.



(c) GUC: the global $\mathcal{G}_{\mathsf{RO}}$ is external to the simulator, and the environment $\mathcal{Z}$ not only has direct access to $\mathcal{G}_{\mathsf{RO}}$, but also invokes arbitraty protocols $\rho_1, \rho_2, \cdots$ alongside the challenge protocol Π.

Figure 2: Comparison of Basic UC, GUC and EUC.

Canetti et al's GUC framework. In Canetti's UC framework, the environment $\mathcal{Z}$ is constrained: it cannot have the direct access to the setup. It means that the setup is not global. This assumption might be impractical in real life applications where it is more plausible that there is a global setup published and used by many protocols.

Motivated by solving problems caused by modeling setup as a local subroutine, Canetti *et al.* introduced Generalized UC (GUC) which can be used for properly analyzing concurrent execution of protocols in the presence of global setup [CDPW07]. In the GUC framework, the environment $\mathcal{Z}$ is unconstrained: $\mathcal{Z}$ is allowed to access the setup directly without going through the simulator/adversary and invoke arbitrary protocols alongside the challenge protocol. Furthermore, the setup can be modeled as a *shared functionality* that can communicate with more than one protocol sessions. Let the output of the unconstrained PPT environment $\mathcal{Z}$ in the real world (resp. ideal world) execution be denoted by $\mathsf{GEXEC}_{\Pi,\mathcal{A},\mathcal{Z}}$ (resp. $\mathsf{GEXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$).

Definition 3. *We say a protocol* Π *GUC-realizes functionality* $\mathcal{F}$*, if for any unconstrained PPT environment* $\mathcal{Z}$ *and any PPT adversary* $\mathcal{A}$ *there exists a PPT simulator* $\mathcal{S}$ *s.t.* $\mathsf{GEXEC}_{\Pi,\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{GEXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

Since the unconstrained environment $\mathcal{Z}$ is given a high-level of flexibility: $\mathcal{Z}$ is allowed to invoke arbitrary protocols in parallel with the challenge protocol. This makes it extremely hard to prove the GUC security. Therefore, a simplified framework called Externalized UC (EUC) is introduced in [CDPW07]. In the EUC framework, the environment $\mathcal{Z}$ has direct access to the shared functionality $\mathcal{G}$ but does not initiate any new protocol sessions except the challenge protocol session. We call such an environment is $\mathcal{G}$-externalized constrained. We say a protocol Π is $\mathcal{G}$-subroutine respecting if it only shares state information via a single shared functionality $\mathcal{G}$. We take RO models as an example, and present the comparison of basic UC, GUC and EUC in Figure 2.

Definition 4. *Let the protocol* Π *be* $\mathcal{G}$*-subroutine respecting. We say a protocol* Π *EUC-realizes*

functionality $\mathcal{F}$ with respect to shared functionality $\mathcal{G}$, if for any PPT $\mathcal{G}$-externalized constrained environment $\mathcal{Z}$ and any PPT adversary $\mathcal{A}$ there exists a PPT simulator $\mathcal{S}$ s.t. $\mathsf{EXEC}^{\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

In [CDPW07], Canetti *et al.* showed that for any $\mathcal{G}$-subroutine respecting protocol Π, proving Π EUC-realizes $\mathcal{F}$ with respect to $\mathcal{G}$ is equivalent to proving Π GUC-realizes $\mathcal{F}$. Therefore, when we want to prove the GUC security of a protocol, we always turn to EUC security for the sake of simplicity.

2.3 The Global Random Oracle Models

In this section, we review four well-known Global Random Oracle (GRO) models: (i) Global Strict Random Oracle (GSRO) model proposed by Canetti *et al.* in [CJS14], which does not give any extra power to anyone; (ii) Global Observable Random Oracle (GORO) model[2] proposed by Canetti *et al.* in [CJS14], which grants the ideal world simulator access to the list of illegitimate queries (to be defined later); (iii) Global Programmable Random Oracle (GPRO) model proposed by Camenisch *et al.* in [CDG+18], which allows the simulator/adversary to program on unqueried points; (iv) Global Programmable and Observable Random Oracle (GPORO) model proposed by Camenisch *et al.* in [CDG+18], which provides both programmability and observability. We present the relation of these models in Figure 3, and the formal description of all the global random oracle models mentioned above in Figure 4.

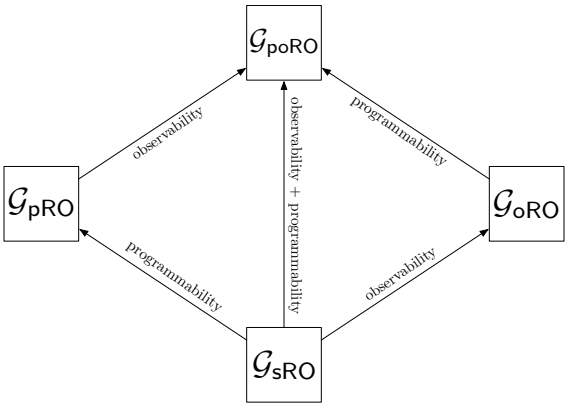


Figure 3: Relation of the Global Random Oracle Models

The GSRO model. The GSRO model $\mathcal{G}_{\mathsf{sRO}}$ is a natural extension of local RO model $\mathcal{F}_{\mathsf{RO}}$: as depicted in Figure 4(a), upon receiving $(\textsc{Query}, \mathsf{sid}, x)$ from any party, $\mathcal{G}_{\mathsf{sRO}}$ first checks if the query (sid, x) has been queried before. If not, $\mathcal{G}_{\mathsf{sRO}}$ answers with a random value of pre-specified length, that is $v \in \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$, and records the tuple (sid, x, v); otherwise, the previously chosen value v is returned again even if the earlier query was made by another party. The sad truth is that Canetti *et al.* remarked that $\mathcal{G}_{\mathsf{sRO}}$ does not suffice to GUC-realizes commitment functionality. Therefore, stronger variant global random oracle models are needed to realize non-trivial functionalities.

[2] In [CDG+18], Camenisch *et al.* used the notations Restricted Observable Global Random oracles (GroRO), Restricted Programmable Global Random Oracles (GrpRO) and Restricted Observable and Programmable Global Random Oracles (GrpoRO). Here we adopt the notations GORO, GPRO and GPORO which skips the "r" for the sake of the simplicity as in [CSW20].

Shared Functionality $\mathcal{G}_{\mathsf{sRO}}$

The functionality interacts with a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ and an adversary $\mathcal{S}$. It is parameterized by the input/output length $\ell_{\mathsf{in}}(\lambda)$ and $\ell_{\mathsf{out}}(\lambda)$. It maintains an initially empty list List.

- **Query.** Upon receiving $(\textsc{Query}, s, x)$ from a party $P_i \in \mathcal{P}$ where $P_i = (\mathsf{pid}, \mathsf{sid})$, or the adversary $\mathcal{S}$:
  - Find v such that $(s, x, v) \in \mathsf{List}$. If there is no such v exists, select an uniformly random $v \in \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$ and record the tuple (s, x, v) in List.
  - Return $(\textsc{QueryConfirm}, s, v)$ to the requestor.

(a) The Global Strict Random Oracle Model $\mathcal{G}_{\mathsf{sRO}}$

Shared Functionality $\mathcal{G}_{\mathsf{oRO}}$

The functionality interacts with a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ and an adversary $\mathcal{S}$. It is parameterized by the input/output length $\ell_{\mathsf{in}}(\lambda)$ and $\ell_{\mathsf{out}}(\lambda)$, and a list of ideal functionality programs $\bar{\mathcal{F}}$. It maintains an initially empty list List.

- **Query.** Same as $\mathcal{G}_{\mathsf{sRO}}$ depicted in Figure 4(a), except when $\mathsf{sid} \neq s$, add the tuple (s, x, v) to the (initially empty) list of illegitimate queries for SID s, which we denote by $\mathcal{Q}_s$.
- **Observe.** Upon receiving a request from an instance of an ideal functionality in the list $\bar{\mathcal{F}}$, with SID s, return to this instance the list of illegitimate queries $\mathcal{Q}_s$ for SID s.

(b) The Global Observable Random Oracle Model $\mathcal{G}_{\mathsf{oRO}}$

Shared Functionality $\mathcal{G}_{\mathsf{pRO}}$

The functionality interacts with a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ and an adversary $\mathcal{S}$. It is parameterized by the input/output length $\ell_{\mathsf{in}}(\lambda)$ and $\ell_{\mathsf{out}}(\lambda)$. It maintains initially empty lists $\mathsf{List}, \mathsf{Prog}$.

- **Query.** Same as $\mathcal{G}_{\mathsf{sRO}}$ depicted in Figure 4(a).
- **Program.** Upon receiving $(\textsc{Program}, \mathsf{sid}, x, v)$ with $v \in \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$ from an adversary $\mathcal{S}$:
  - If $\exists v' \in \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$ such that $(\mathsf{sid}, x, v') \in \mathsf{List}$ and $v \neq v'$, ignore this input.
  - Set $\mathsf{List} := \mathsf{List} \cup \{(\mathsf{sid}, x, v)\}$ and $\mathsf{Prog} := \mathsf{Prog} \cup \{(\mathsf{sid}, x)\}$.
  - Return $(\textsc{ProgramConfirm}, \mathsf{sid})$ to $\mathcal{S}$.
- **IsPrograMed.** Upon receiving $(\textsc{IsPrograMed}, \mathsf{sid}', x)$ from a party P_i or the adversary $\mathcal{S}$:
  - If the input was given by $P_i = (\mathsf{pid}, \mathsf{sid})$ and $\mathsf{sid} \neq \mathsf{sid}'$, ignore this input.
  - If $(\mathsf{sid}', x) \in \mathsf{Prog}$, set $b := 1$; otherwise, set $b := 0$.
  - Return $(\textsc{IsPrograMed}, \mathsf{sid}', b)$ to the requester.

(c) The Global Programmable Random Oracle Model $\mathcal{G}_{\mathsf{pRO}}$

Shared Functionality $\mathcal{G}_{\mathsf{poRO}}$

The functionality interacts with a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ and an adversary $\mathcal{S}$. It is parameterized by the input/output length $\ell_{\mathsf{in}}(\lambda)$ and $\ell_{\mathsf{out}}(\lambda)$, and a list of ideal functionality programs $\bar{\mathcal{F}}$. It maintains initially empty lists $\mathsf{List}, \mathsf{Prog}$.

- **Query/Observe.** Same as $\mathcal{G}_{\mathsf{oRO}}$ depicted in Figure 4(b).
- **Program/IsPrograMed.** Same as $\mathcal{G}_{\mathsf{pRO}}$ depicted in Figure 4(c).

(d) The Global Programmable and Observable Random Oracle Model $\mathcal{G}_{\mathsf{poRO}}$

Figure 4: The Global Random Oracle Models.

The GORO model. Compared to $\mathcal{G}_{\mathsf{sRO}}$, the GORO model $\mathcal{G}_{\mathsf{oRO}}$ provides additionally observability. More precisely, some of the queries can be marked as "illegitimate" and potentially disclosed to the simulator. As depicted in Figure 4(b), the GORO functionality $\mathcal{G}_{\mathsf{oRO}}$ interacts with a list of ideal functionality programs $\bar{\mathcal{F}} = \{\mathcal{F}_1, \ldots, \mathcal{F}_n\}$, where $\mathcal{F}_1, \ldots, \mathcal{F}_n$ are the protocol functionalities (e.g., commitment functionality, ZK functionality, etc.) that share the same global setup $\mathcal{G}_{\mathsf{oRO}}$. For any query (s, x) from any party $P = (\mathsf{pid}, \mathsf{sid})$ where s is the content of the SID field, if $s \neq \mathsf{sid}$, then this query is considered "illegitimate". After that, $\mathcal{G}_{\mathsf{oRO}}$ adds the tuple (s, x, v) to the list of illegitimate queries for SID s, which we denote as $\mathcal{Q}_s$. The illegitimate queries $\mathcal{Q}_s$ may be disclosed to an instance of ideal functionality $\mathcal{F}_i \in \bar{\mathcal{F}}$ whose SID is the one of the illegitimate queries. Then the ideal functionality instance $\mathcal{F}_i$ leaks the illegitimate queries to the simulator.

The GPRO model. Compared to $\mathcal{G}_{\mathsf{sRO}}$, the GPRO model $\mathcal{G}_{\mathsf{pRO}}$ additionally allows simulator/adversary to program the global random oracle on unqueried points. As depicted in Figure 4(c), upon receiving $(\textsc{Program}, \mathsf{sid}, x, v)$ from the simulator/adversary, $\mathcal{G}_{\mathsf{pRO}}$ first checks if (sid, x) has been queried before. If not, $\mathcal{G}_{\mathsf{pRO}}$ stores (sid, x, v) in the query-answer lists. Any honest party can check whether a point has been programmed or not by sending the $(\textsc{IsProgramed}, \mathsf{sid}, x)$ command to $\mathcal{G}_{\mathsf{pRO}}$. Thus, in the real world, the programmed points can always be detected. However, in the ideal world, the simulator $\mathcal{S}$ can successfully program the random oracle without being detected since it can always return $(\textsc{IsProgramed}, \mathsf{sid}, 0)$ when the adversary invokes $(\textsc{IsProgramed}, \mathsf{sid}, x)$ to verify whether a point x has been programmed or not.

The GPORO model. If we combine the GORO model and GPRO model together, we obtain the GPORO model $\mathcal{G}_{\mathsf{poRO}}$ which is depicted in Figure 4(d). To the best of our knowledge, the GPORO model is the most powerful GRO model that enables efficient composable protocols in the GUC framework. For example, Camenisch *et al.* gave an efficient non-interactive GUC-secure commitment protocol in the GPORO model [CDG+18].

Remark 1. *Camenisch et al. remarked that when one uses the (distinguishing) environment in a cryptographic reduction, one can have full control over the shared functionality [CDG+18]. More precisely, as depicted in Figure 5, the reduction algorithm $\mathcal{B}$ simulates the complete view of the environment $\mathcal{Z}$ including the shared functionality $\mathcal{G}$, thus $\mathcal{B}$ has full control of $\mathcal{G}$.*

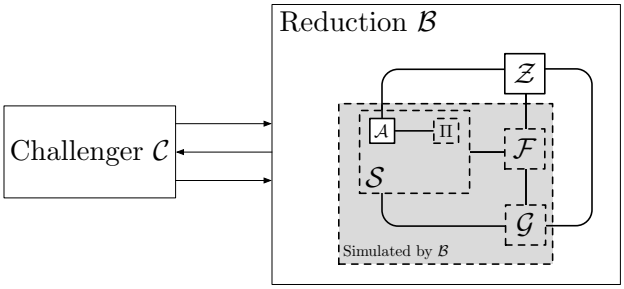


Figure 5: In order to play against the external challenger $\mathcal{C}$, reduction algorithm $\mathcal{B}$ simulates everything (marked as gray) including shared functionality $\mathcal{G}$, starts the protocol Π with the real world adversary $\mathcal{A}$/environment $\mathcal{Z}$ by running $\mathcal{A}/\mathcal{Z}$ internally as black-box.

2.4 One-Way Functions

One-Way Function (OWF) is the minimal cryptographic primitive, and we take this definition from [KL20]. Informally, a one-way function $g : \{0,1\}^\lambda \to \{0,1\}^\lambda$ is: (i) easy to compute, (ii) but

hard to invert. The first requirement is easy to formalize: we require that g to be computable in polynomial time. In order to formalize the second requirement, we consider the following experiment:

Experiment $\text{EXPT}^{\text{HtI}}_{\mathcal{A},g}(\lambda)$:

1. Choose a uniformly random $x \leftarrow \{0,1\}^\lambda$ and compute $y := g(x)$.
2. $\mathcal{A}$ is given 1^λ and y as input, and it outputs x'.
3. If $y = g(x')$, output 1; otherwise, output 0.

Here we denote by $\mathbf{Adv}^{\text{HtI}}_{\mathcal{A},g}(\lambda) := \Pr[\text{EXPT}^{\text{HtI}}_{\mathcal{A},g}(\lambda) = 1]$ the advantage of $\mathcal{A}$.

Now we define what it means for a function g to be one-way.

Definition 5. *We say a function $g : \{0,1\}^\lambda \to \{0,1\}^\lambda$ is one-way if the following two conditions holds:*

1. ***(Easy to Compute)*** *We say it is* easy to compute *if there exists a PPT algorithm M_g computing g; that is, $z := M_g(x)$ and $z = g(x)$ for all x.*
2. ***(Hard to Invert (*HtI*))*** *For any PPT adversary $\mathcal{A}$, we say it is hard to invert if there exists a negligible function* negl *such that $\mathbf{Adv}^{\text{HtI}}_{\mathcal{A},g}(\lambda) \leq \mathsf{negl}(\lambda)$.*

2.5 SHVZK Protocols

A 3-round public coin Special Honest Verifier Zero-Knowledge (SHVZK) protocol $\Pi = \Pi.\{\mathsf{Move1}, \mathsf{Move2}, \mathsf{Move3}, \mathsf{Verify}, \mathsf{Sim}\}$ allows a prover to convince a verifier that a statement x is true with the aid of the witness w. In the first round, the prover computes and sends the first flow message $a := \mathsf{Move1}(x, w; r)$ using the statement-witness pair (x, w) and some random coin r. In the second round, the verifier samples and sends a uniformly random public coin challenge $e \leftarrow \mathsf{Move2}(1^\lambda)$. In the last round, the prover computes the response to the challenge $z := \mathsf{Move3}(x, w, e; r)$ using the statement-witness pair (x, w), challenge e and the random coin r. Finally the verifier accepts the statement x if and only if $\mathsf{Verify}(x, a, e, z) = 1$. We put the workflow of the SHVZK protocol in Figure 6. We often call (a, e, z) the transcript between the prover and the verifier.

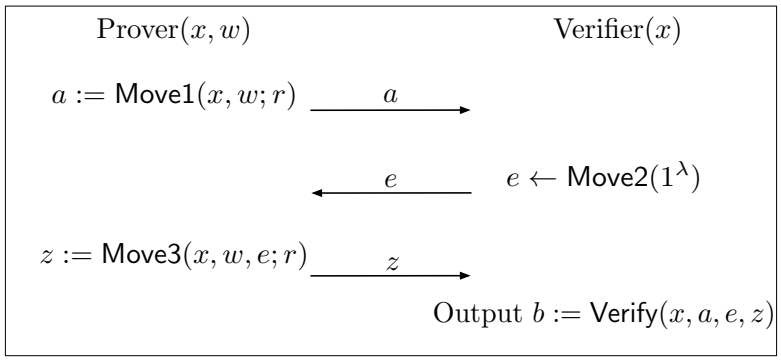


Figure 6: The Workflow of the SHVZK Protocol

A SHVZK protocol should satisfy (i) perfect completeness, i.e. any honest prover who holds the witness w such that $(x, w) \in \mathcal{R}_\mathcal{L}$ can always make the verifier accept; (ii) k-special soundness,

i.e. given any k distinct accepting transcripts, we can always extract the witness w; (iii) Special Honest Verifier Zero-Knowledge (SHVZK) property, i.e. given the challenge e ahead, there should be a PPT simulator algorithm Sim that takes the statement x, the challenge e and random coin r as input, and outputs the simulated (a, z) which is indistinguishable from the real one. The first property is easy to formalize. In order to formalize the k-special soundness, we consider the following experiment:

Experiment $\text{EXPT}^{\text{k-SS}}_{\mathcal{A},\Pi}(\lambda)$:

1. $\mathcal{A}$ outputs a statement x along with k transcripts $\{(a, e_i, z_i)\}_{i\in[k]}$.
2. If $e_i \neq e_j$ where $i \neq j$: extract the witness w' from $\{(a, e_i, z_i)\}_{i\in[k]}$
3. If $(x, w') \in \mathcal{R}_{\mathcal{L}}$, output 1; otherwise, output 0.

Here we denote by $\mathbf{Adv}^{\text{k-SS}}_{\mathcal{A},\Pi}(\lambda) := \Pr[\text{EXPT}^{\text{k-SS}}_{\mathcal{A},\Pi}(\lambda) = 1]$ the advantage of $\mathcal{A}$.

We define the SHVZK property via the following experiment:

Experiment $\text{EXPT}^{\text{SHVZK}}_{\mathcal{A},\Pi}(\lambda)$:

1. $\mathcal{A}$ outputs a statement-witness pair (x, w) along with a challenge e.
2. If $(x, w) \in \mathcal{R}_{\mathcal{L}}$: select a random string r and a random bit $b \in \{0, 1\}$,

   and compute the following:

   (a) If $b = 0$: $a := \mathsf{Move1}(x, w; r)$; $z := \mathsf{Move3}(x, w, e; r)$.

   (b) If $b = 1$: $(a, z) := \mathsf{Sim}(x, e; r)$.
3. $\mathcal{A}$ is given (a, z) as input, and it outputs a guess bit $b' \in \{0, 1\}$.
4. If $b = b'$, output 1; otherwise, output 0.

Here we denote by $\mathbf{Adv}^{\text{SHVZK}}_{\mathcal{A},\Pi}(\lambda) := \left|\Pr[\text{EXPT}^{\text{SHVZK}}_{\mathcal{A},\Pi}(\lambda) = 1] - \frac{1}{2}\right|$ the advantage of $\mathcal{A}$.

Now we can formally define the SHVZK protocol.

Definition 6. *We say a protocol* $\Pi = \Pi.\{\mathsf{Move1}, \mathsf{Move2}, \mathsf{Move3}, \mathsf{Verify}, \mathsf{Sim}\}$ *is a SHVZK protocol if the following condition holds:*

1. ***(Perfect Completeness)*** *For any* $(x, w) \in \mathcal{R}_{\mathcal{L}}$*, we say it is perfect complete*

$$\Pr\left[a := \mathsf{Move1}(x, w; r); e \leftarrow \mathsf{Move2}(1^\lambda); z := \mathsf{Move3}(x, w, e; r) : \mathsf{Verify}(x, a, e, z) = 1\right] = 1$$

2. ***(k-Special Soundness)*** *For any PPT adversary* $\mathcal{A}$*, we say it has* k*-special soundness where* $k \in \mathbb{N}$ *and* $k \geq 2$*, if there exists a negligible function* negl *such that* $\mathbf{Adv}^{\text{k-SS}}_{\mathcal{A},\Pi}(\lambda) \leq \mathsf{negl}(\lambda)$.

3. ***(Special Honest Verifier Zero-Knowledge)*** *We say it has SHVZK if there exists a PPT simulator* Sim *such that for any PPT adversary* $\mathcal{A}$*, there exists a negligible function* negl *such that* $\mathbf{Adv}^{\text{SHVZK}}_{\mathcal{A},\Pi}(\lambda) \leq \mathsf{negl}(\lambda)$.

2.6 Non-Interactive Witness Hiding Argument

2.6.1 Non-Interactive Witness Hiding Argument in the Plain Model

Witness Hiding (WH) interactive proofs were introduced by Feige and Shamir in [FS90], and we employ the Non-Interactive Witness-Hiding (NIWH) argument here [KZ20]. A Non-Interactive Witness-Hiding (NIWH) argument $\Pi = \Pi.\{\mathsf{Prove}, \mathsf{Verify}\}$ allows the prover to generate the proof π using the statement-witness pair (x, w) and a random string r and send π to the verifier. The verifier checks if the proof π is valid and outputs a bit b indicating the acceptance or rejection. Formally, the NIWH argument has the following algorithms:

- $\pi := \mathsf{Prove}(x, w; r)$ takes input as a statement-witness pair (x, w) and a random string r, and it outputs a proof π. When r is not important, we use $\mathsf{Prove}(x, w)$ for simplicity.
- $b := \mathsf{Verify}(x, \pi)$ takes input as a statement x and a proof π, and it outputs a bit b indicating acceptance or rejection.

Basically, the NIWH argument should satisfy the perfect completeness and computational soundness. The perfect completeness is trivial. The computational soundness means that any PPT prover cannot convince the verifier that a false statement is true with overwhelming probability. the NIWH proof/argument should satisfy the witness hiding property: given the proof π generated by the prover, the verifier cannot compute any new witness that the verifier does not know before the interaction. Formally, we first consider the following experiment:

Experiment $\text{EXPT}^{\text{WH}}_{\mathcal{A},\Pi}(\lambda)$:

1. Select $(x, w) \in \mathcal{R}_{\mathcal{L}}$, and compute $\pi \leftarrow \mathsf{Prove}(x, w)$.
2. $\mathcal{A}$ is given (x, π) as input, and it outputs w'.
3. If $(x, w') \in \mathcal{R}_{\mathcal{L}}$, output 1; otherwise, output 0.

Here we denote by $\mathbf{Adv}^{\text{WH}}_{\mathcal{A},\Pi}(\lambda) := \Pr[\text{EXPT}^{\text{WH}}_{\mathcal{A},\Pi}(\lambda) = 1]$ the advantage of $\mathcal{A}$.

Before giving the formal definition of the NIWH argument, we have to define the hard instance ensembles, as in [Pas03].

Definition 7 (Hard Instance Ensembles)**.** *Let $\mathcal{R}_{\mathcal{L}}$ be an NP relation, and $\mathcal{L}$ be its associate language, and $\mathcal{X} = \{\mathcal{X}_\lambda\}_{\lambda\in\mathbb{N}}$ be a probability ensemble s.t. $\mathcal{X}_\lambda$ ranges over $\mathcal{L} \cap \{0,1\}^\lambda$. We say that $\mathcal{X}$* is hard for NP relation $\mathcal{R}_{\mathcal{L}}$ *if for any PPT $\mathcal{A}$ and any $x \in \mathcal{X}$, there exists a negligible function* negl *s.t.* $\Pr[(x, \mathcal{A}(x)) \in \mathcal{R}_{\mathcal{L}}] = \mathsf{negl}(\lambda)$.

Now we can formally define the NIWH argument.

Definition 8. *Fix an NP relation $\mathcal{R}_{\mathcal{L}}$ whose associate language is $\mathcal{L}$. We say a protocol $\Pi = \Pi.\{\mathsf{Prove}, \mathsf{Verify}\}$ is a NIWH argument for $\mathcal{R}_{\mathcal{L}}$ if the following conditions hold:*

1. ***(Perfect Completeness)*** *For any $(x, w) \in \mathcal{R}_{\mathcal{L}}$, we say it is perfect complete if*

$$\Pr[\pi \leftarrow \mathsf{Prove}(x, w) : \mathsf{Verify}(x, \pi) = 1] = 1$$

2. ***(Computational Soundness)*** *For any $x \notin \mathcal{L}$, we say it is computational sound if for any PPT adversary $\mathcal{A}$, there exists a negligible function* negl *such that*

$$\Pr[\pi^* \leftarrow \mathcal{A}(x) : \mathsf{Verify}(x, \pi^*) = 1] \leq \mathsf{negl}(\lambda)$$

3. ***(Witness Hiding)*** *Let $\mathcal{X} = \{\mathcal{X}_\lambda\}_{\lambda\in\mathbb{N}}$ be a hard instance ensemble for $\mathcal{R}_\mathcal{L}$. We say it is witness hiding for $\mathcal{R}_\mathcal{L}$ under the instance ensemble $\mathcal{X}$ if for any PPT adversary $\mathcal{A}$ and any $(x, w) \in \mathcal{R}_\mathcal{L}$ with $x \in \mathcal{X}$, there exists a negligible function negl such that $\mathbf{Adv}^{\mathrm{WH}}_{\mathcal{A},\Pi}(\lambda) \leq \mathsf{negl}(\lambda)$. We say it is witness hiding for $\mathcal{R}_\mathcal{L}$ if it is witness hiding under all hard-instance ensembles $\mathcal{X}$ for $\mathcal{R}_\mathcal{L}$.*

2.6.2 Non-Interactive Witness Hiding Argument in the Random Oracle Model

We here discuss the NIWH argument in the random oracle model. Note that, stronger security property such as *(straight-line) extractability* can now be achieved in the random oracle model: an extraction algorithm Ext could be constructed to extract the witness from a maliciously generated proof. More concretely, in a NIWH argument in the random oracle model $\Pi = \Pi.\{\mathsf{Prove}^\mathcal{O}, \mathsf{Verify}^\mathcal{O}, \mathsf{Ext}\}$, both the prover and the verifier are allowed to query the random oracle $\mathcal{O}$ at any moment, during the protocol execution. As in the plain model, the prover generates the proof π using the statement-witness pair (x, w) and a random string r and sends π to the verifier, and the verifier then verifies if the proof π is valid or not; the verifier outputs a bit b indicating the acceptance or rejection. Formally, the Prove and Verify algorithms in a NIWH argument in the random oracle model are described as follows:

- $\pi := \mathsf{Prove}^\mathcal{O}(x, w; r)$ takes input as a statement-witness pair (x, w) and a random string r, and it is allowed to query the random oracle $\mathcal{O}$. It outputs a proof π. When r is not important, we use $\mathsf{Prove}^\mathcal{O}(x, w)$ for simplicity.
- $b := \mathsf{Verify}^\mathcal{O}(x, \pi)$ takes input as a statement x and a proof π, and it is allowed to query the random oracle $\mathcal{O}$. It outputs a bit b indicating acceptance or rejection.

The basic properties, such as perfect completeness, computational soundness and witness-hiding, can be defined for a NIWH argument in the random oracle model as that in the plain model above. We now describe how to define the straight-line extractability property; note that our extractability definition is taken from that by Pass [Pas03]. To enable the extractability, typically, the extraction algorithm Ext can be developed by simulating the random oracle for the prover and the verifier, and thus the algorithm Ext has full control of the random oracle. In this paper, we consider a much more restricted random oracle, and the algorithm Ext is granted only with the *observability*; that is, Ext is allowed to see the query-answer list of the random oracle. For that reason, we write $\mathsf{Ext}^\mathcal{O}$ to indicate that, the extraction algorithm Ext does not have the full control of the random oracle, and is only granted to have the observability capability.

Definition 9 (Straight-line Extractability in the Random Oracle Model)**.** *Fix an NP relation $\mathcal{R}_\mathcal{L}$ whose associate language is $\mathcal{L}$. Consider random oracle $\mathcal{O}$, and a NIWH argument for $\mathcal{R}_\mathcal{L}$ in the random oracle model $\Pi = \Pi.\{\mathsf{Prove}^\mathcal{O}, \mathsf{Verify}^\mathcal{O}, \mathsf{Ext}^\mathcal{O}\}$. For any $x \in \mathcal{L}$, we say the NIWH argument Π is* straight-line extractable *if for any PPT adversary $\mathcal{A}$,*

$$\Pr[\pi^* \leftarrow \mathcal{A}^\mathcal{O}(x); b := \mathsf{Verify}^\mathcal{O}(x, \pi^*); w^* \leftarrow \mathsf{Ext}^\mathcal{O}(x, \pi^*) \quad : \quad b = 1 \ \wedge \ (x, w^*) \in \mathcal{R}_\mathcal{L}] \geq 1 - \mathsf{negl}(\lambda)$$

We call the probability of $\mathsf{Ext}^\mathcal{O}$ failing to extract the valid witness from any accepting proof π the knowledge error. From the definition above, it is easy to see that the knowledge error of $\mathsf{Ext}^\mathcal{O}$ should be negligible.

2.7 Equivocal Commitment

An equivocal commitment scheme $\Pi = \Pi.\{\mathsf{KeyGen}, \mathsf{KeyVer}, \mathsf{Commit}, \mathsf{ComVer}, \mathsf{EquCom}, \mathsf{Equiv}\}$ allows the committer to generate the commitment c to any value m using the commitment key ck and the randomness r. Later, the committer can open c to m by sending the the opening d to the receiver who verifies it. Furthermore, if the committer obtains the trapdoor td with respect to the ck, he can generate the equivocal commitment $\tilde{c}$, later open $\tilde{c}$ to any message $\tilde{m}$. Formally, the equivocal commitment has the following algorithms:

- $(\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda)$ takes input as the security parameter λ, and outputs a commitment key ck and the trapdoor td.
- $b := \mathsf{KeyVer}(\mathsf{ck}, \mathsf{td})$ takes input as a commitment key ck and a trapdoor td. It outputs a bit b indicating acceptance or rejection.
- $(c, d) := \mathsf{Commit}(\mathsf{ck}, m; r)$ takes input as a commitment key ck, a message m and a randomness r. It outputs the commitment c and the opening d. We assume that there exists a deterministic algorithm that can extract m from d. When r is not important, we use $\mathsf{Commit}(\mathsf{ck}, m)$ for simplicity.
- $b := \mathsf{ComVer}(\mathsf{ck}, c, d)$ takes input as a commitment key ck, and a commitment-opening pair (c, d). It outputs a bit b indicating acceptance or rejection.
- $(\tilde{c}, \mathsf{st}) := \mathsf{EquCom}(\mathsf{ck}, \mathsf{td}; r)$ takes input as a commitment key ck, a trapdoor td, and a randomness r. It outputs a commitment $\tilde{c}$ and a state st. When r is not important, we use $\mathsf{EquCom}(\mathsf{ck}, \mathsf{td})$ for simplicity.
- $\tilde{d} := \mathsf{Equiv}(\mathsf{ck}, \mathsf{td}, \tilde{c}, \mathsf{st}, \tilde{m})$ takes input as a commitment key ck, a trapdoor td, a commitment $\tilde{c}$, a state st, and an arbitrary message $\tilde{m}$ for which equivocation is required. It outputs an opening $\tilde{d}$.

The equivocal commitment requires the following properties: correctness, hiding, binding and equivocation. Correctness means that any commitment produced by the honest committer can always be verified. Hiding means that the commitment c reveals nothing about the message m. This is defined via the following experiment:

Experiment $\textsc{expt}^{\text{hiding}}_{\mathcal{A},\Pi}(\lambda)$:

1. Run $(\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda)$.
2. $\mathcal{A}$ is given ck as input, and it outputs two distinct messages m_0, m_1.
3. Select a random bit $b \in \{0, 1\}$, and compute $c \leftarrow \mathsf{Commit}(\mathsf{ck}, m_b)$.
4. $\mathcal{A}$ is given m_b as input, and it outputs a bit b'.
5. If $b' = b$, output 1; otherwise, output 0.

Here we denote by $\mathbf{Adv}^{\text{hiding}}_{\mathcal{A},\Pi}(\lambda) := \left|\Pr[\textsc{expt}^{\text{hiding}}_{\mathcal{A},\Pi}(\lambda) = 1] - \frac{1}{2}\right|$ the advantage of $\mathcal{A}$.

Binding means that it is infeasible for the committer to output the commitment c that can be opened in two different ways.

Experiment $\text{EXPT}^{\text{binding}}_{\mathcal{A},\Pi}(\lambda)$:

1. Run $(\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda)$.
2. $\mathcal{A}$ is given ck as input, and it outputs (c, d_0, d_1).
3. If $d_0 \neq d_1$ and $\mathsf{ComVer}(\mathsf{ck}, c, d_0) = \mathsf{ComVer}(\mathsf{ck}, c, d_1) = 1$ holds,

   output 1; otherwise, output 0.

Here we denote by $\mathbf{Adv}^{\text{binding}}_{\mathcal{A},\Pi}(\lambda) := \Pr[\text{EXPT}^{\text{binding}}_{\mathcal{A},\Pi}(\lambda) = 1]$ the advantage of $\mathcal{A}$.

Equivocation means that given the trapdoor td, one can open a previously constructed commitment c of message m to other message $\tilde{m} \neq m$.

Experiment $\text{EXPT}^{\text{equivocal}}_{\mathcal{A},\Pi}(\lambda)$:

1. $\mathcal{A}$ is given 1^λ as input, and it outputs $(\mathsf{ck}, \mathsf{td}, m)$.
2. If $\mathsf{KeyVer}(\mathsf{ck}, \mathsf{td}) = 1$: select a random string r and a random bit $b \in \{0, 1\}$,

   and compute the following:

   (a) If $b = 0$: invoke $(c, d) := \mathsf{Commit}(\mathsf{ck}, m; r)$.

   (b) If $b = 1$: invoke $(c, \mathsf{st}) := \mathsf{EquCom}(\mathsf{ck}, \mathsf{td}; r); d := \mathsf{Equiv}(\mathsf{ck}, \mathsf{td}, \tilde{c}, \mathsf{st}, m)$.
3. $\mathcal{A}$ is given (c, d) as input, and it outputs a bit b'.
4. If $b = b'$, output 1; otherwise, output 0.

Here we denote by $\mathbf{Adv}^{\text{equivocal}}_{\mathcal{A},\Pi}(\lambda) := \left|\Pr[\text{EXPT}^{\text{equivocal}}_{\mathcal{A},\Pi}(\lambda) = 1] - \frac{1}{2}\right|$ the advantage of $\mathcal{A}$.

Now we can formally define the equivocal commitment, and it should satisfy the following definition:

Definition 10. *We say a scheme* $\Pi = \Pi.\{\mathsf{KeyGen}, \mathsf{KeyVer}, \mathsf{Commit}, \mathsf{ComVer}, \mathsf{EquCom}, \mathsf{Equiv}\}$ *is an equivocal commitment if the following conditions hold:*

1. ***(Perfect Correctness)*** *For any message m, we say it is perfect correct if*

$$\Pr[(\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda); (c, d) \leftarrow \mathsf{Commit}(\mathsf{ck}, m) : \mathsf{ComVer}(\mathsf{ck}, c, d) = 1] = 1$$

2. ***(Perfect Hiding)*** *We say it is perfect hiding if for any adversary $\mathcal{A}$ s.t.* $\mathbf{Adv}^{\text{hiding}}_{\mathcal{A},\Pi}(\lambda) = 0$.
3. ***(Computational Binding)*** *We say it is computational binding if for any PPT adversary $\mathcal{A}$, there exists a negligible function* negl *s.t.* $\mathbf{Adv}^{\text{binding}}_{\mathcal{A},\Pi}(\lambda) \leq \mathsf{negl}(\lambda)$.
4. ***(Equivocation)*** *We say it is equivocal if for any PPT adversary $\mathcal{A}$, there exists a negligible function* negl *s.t.* $\mathbf{Adv}^{\text{equivocal}}_{\mathcal{A},\Pi}(\lambda) \leq \mathsf{negl}(\lambda)$.

2.8 "MPC-in-the-Head" Paradigm

In [IKOS07], Ishai *et al.* proposed the famous "MPC-in-the-head" paradigm from which we can construct a SHVZK protocol using the MPC protocol. Before introducing the details of the paradigm, we have to define the MPC protocol first.

Consider a function $f : (\{0,1\}^\lambda)^{n+1} \to \{0,1\}^\lambda$. We let $P_1, \ldots, P_n$ be n parties modeled as PPT interactive machines. Assume that each party P_i holds a private input $w_i \in \{0,1\}^\lambda$ and a public input $x \in \{0,1\}^\lambda$, and wants to compute $y = f(x, w)$, where $w = (w_1, \ldots, w_n)$. They communicate with each other using point-to-point secure channels (e.g. encrypted channels or OT channels) in the synchronous model. To achieve this goal, the parties jointly run a secure Multi-Party Computation (MPC) protocol Π_{MPC}. The protocol Π_{MPC} is specified via the next-message functions: there are multiple communication rounds, and in each round the party P_i sends into the channel a message that is computed as a deterministic function of the internal state of P_i (including private input w_i and random tape k_i) and the messages that P_i has received in the previous rounds. We denote by $\mathsf{view}_i(x, w_i)$ the view of P_i, which is the concatenation of the inputs x, w_i, the random tape k_i and all the messages received by P_i during the execution of Π_{MPC}. Each secure channel defines a relation of consistency between views. For instance, in the plain model, we say $\mathsf{view}_i(x, w_i)$ and $\mathsf{view}_j(x, w_j)$ are consistent if the outgoing messages in $\mathsf{view}_i(x, w_i)$ are identical to the incoming messages in $\mathsf{view}_j(x, w_j)$ and vice versa. Finally, all the views should yield the same output y, i.e. there are n functions $\Pi_{f,1}, \ldots, \Pi_{f,n}$ such that $y = \Pi_{f,i}(\mathsf{view}_i(x, w_i))$ for all $i \in [n]$. We note that, for our purpose of use, we require that every party P_i in the honest execution of Π_{MPC} has the same output y; while in the general case, the output of P_i can be different from each other.

In this work, we only consider security of MPC protocols in the semi-honest model. In the semi-honest model, the corrupted parties follow the instructions of the protocol, but are curious about the private information of other parties. Thus, the protocol needs to be designed in such a way that a corrupted P_i cannot infer information about w_j from its view $\mathsf{view}_i(x, w_i)$, where $j \neq i$.

We denote by $\mathsf{view}_T(x, w_1, \ldots, w_n)$ the joint view of players in set $T \subset [n]$ for the execution of Π_{MPC} on input $(x, w_1, \ldots, w_n)$. Consider a PPT simulator algorithm Sim that given the set $T \subset [n]$, the output of Π_{MPC} which realizes f on input $(x, w_1, \ldots, w_n)$ (i.e. $f(x, w_1, \ldots, w_n)$), and the input of parties in T (i.e. $(x, (w_i)_{i \in T})$), it can output the simulated joint view of players in set T for the execution of Π_{MPC} on input $(x, w_1, \ldots, w_n)$ which we denote by $\mathsf{Sim}(T, x, (w_i)_{i \in T}, f(x, w_1, \ldots, w_n))$. With these notations, we have the following definition.

Definition 11. *We say an n-party protocol Π_{MPC} realizes f in the semi-honest model, if the following conditions hold:*

1. ***(Perfect Correctness)** For any inputs $x, w = (w_1, \ldots, w_n)$ and any random tape, we say Π_{MPC} realizes f with perfect correctness if $\forall i \in [n] : \Pr[y = \Pi_{f,i}(\mathsf{view}_i(x, w_i))] = 1$.*
2. ***(t-Privacy)** Let $1 \leq t < n$. We say Π_{MPC} realizes f with t-privacy if it is perfect correct and for every set of corrupted parties $T \subset [n]$ satisfying $|T| \leq t$, there exists a PPT simulator Sim such that*

$$\mathsf{view}_T(x, w_1, \ldots, w_n) \equiv \mathsf{Sim}(T, x, (w_i)_{i \in T}, f(x, w_1, \ldots, w_n))$$

Now we can introduce the "MPC-in-the-head" paradigm. Let f be the following $(n+1)$-argument function corresponding to an NP relation $\mathcal{R}_\mathcal{L}$: $f(x, w_1, \ldots, w_n) = \mathcal{R}_\mathcal{L}(x, w_1 \oplus \cdots \oplus w_n)$. Here x is a public input known to all parties, w_i is the private input of party P_i, and the output is received by all parties. In a high-level description, the main idea is for the prover to simulate the execution of a t-private n-party MPC protocol that realizes f. Then the prover employs a statically binding commitment to commit to all views of the parties and sends them to the verifier. After that, the verifier chooses a random subset of the parties, where the size of the subset equals t, and asks the prover to open their corresponding views. Finally the verifier accepts the statement if and only if (i) the commitment is correctly opened and (ii) the received views are consistent with each other. We refer interesting readers to see more details in [IKOS07].

3 Impossibility in the GORO Model

In this section, we show that it is impossible to construct 2-round GUC-secure commitment (one round for the committing phase and one round for the opening phase) in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world against static adversaries. We first provide the formal description of transferable commitment functionality $\mathcal{F}_{\mathsf{tCOM}}$ from [CJS14] in Figure 7. The main difference with the traditional commitment functionality is that in $\mathcal{F}_{\mathsf{tCOM}}$, the simulator can request the list of the illegitimate queries from $\mathcal{F}_{\mathsf{tCOM}}$. If we use the traditional commitment functionality which has no such power in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world, the simulator will have no advantage over others at all. This is one of the reasons why transferable ideal functionalities were designed in the presence of the $\mathcal{G}_{\mathsf{oRO}}$ model, and we refer interesting readers to see more discussions in [CJS14].

Functionality $\mathcal{F}_{\mathsf{tCOM}}$

The functionality interacts with two parties C, R and an adversary $\mathcal{S}$.

- Upon receiving $(\textsc{Commit}, \mathsf{sid}, C, R, m)$ from C, do:
  - Record the tuple (sid, C, R, m), and send $(\textsc{Receipt}, \mathsf{sid}, C, R)$ to R and $\mathcal{S}$.
  - Ignore any subsequent Commit command.
- Upon receiving $(\textsc{Decommit}, \mathsf{sid}, C, R)$ from C, do:
  - If there is a tuple (sid, C, R, m) recorded, send $(\textsc{Decommit}, \mathsf{sid}, C, R, m)$ to R and $\mathcal{S}$, and halt.
  - Otherwise, ignore the message.
- When asked by the adversary $\mathcal{S}$, obtain from $\mathcal{G}_{\mathsf{oRO}}$ the list of illegitimate queries $\mathcal{Q}_{\mathsf{sid}}$ that pertain to SID sid, and send $\mathcal{Q}_{\mathsf{sid}}$ to the adversary $\mathcal{S}$.

Figure 7: The Transferable Functionality $\mathcal{F}_{\mathsf{tCOM}}$ for Commitment

We prove this impossibility by contradiction. Suppose that there exists such a 2-round GUC-secure protocol. Let us first consider the case where the receiver is corrupted, the simulator needs to produce an equivocal commitment without knowing the plaintext in the committing phase, and later open it to any given message (a.k.a. the plaintext) in the opening phase. We observe that the receiver does not need to send any message during the 2-round protocol execution, thus when the receiver is controlled by adversary, the corrupted receiver can delay all its $\mathcal{G}_{\mathsf{oRO}}$ queries until it receives the opening message. In this case, the simulator cannot obtain the illegitimate queries of the corrupted receiver before producing the equivocal commitments, and thus has no advantage over the real world adversary. If the simulator still succeeds to produce the equivocal commitments even if it has no illegitimate queries, then distinctions will be revealed when the adversary performs the following attacks. The adversary corrupts the committer, and instructs the committer to run the simulator algorithm mentioned above to generate the commitment message. In this case, where the committer is corrupted, the receiver/simulator needs to extract the plaintext from this commitment message. However, the entire computation of the commitment message is totally independent of the plaintext, thus with high probability the simulation would fail. Formally, we prove this impossibility through Theorem 1.

Theorem 1. *There exists no terminating 2-round (one round for commitment phase and one round for decommitment phase) protocol* Π *that GUC-realizes* $\mathcal{F}_{\mathsf{tCOM}}$ *depicted in Figure 7 with static security, using only the shared functionality for global observable random oracle* $\mathcal{G}_{\mathsf{oRO}}$.

Proof. Suppose there exists such a protocol Π that GUC-realizes $\mathcal{F}_{\mathsf{tCOM}}$ in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world.

Then there must exist a PPT simulator $\mathcal{S}$ such that $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\mathcal{F}_{\mathsf{tCOM}},\mathcal{S},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\Pi,\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{G}_{\mathsf{oRO}}$-externally constrained environment $\mathcal{Z}$.

In particular, let us first consider the session with SID sid_1, and let $\mathcal{A}$ be a dummy adversary that simply forwards protocol flows between corrupt parties and the environment $\mathcal{Z}$. Let $\mathcal{Z}$ corrupt the receiver R^* at first. Then $\mathcal{Z}$ chooses a random bit $b \in \{0,1\}$ and gives it as the input to the honest committer C. After that, $\mathcal{Z}$ waits for C to send the commitment ψ. Next, $\mathcal{Z}$ lets C reveal the committed value b'. If $b = b'$, $\mathcal{Z}$ outputs 1; otherwise, $\mathcal{Z}$ outputs 0.

In order to make the GUC experiments above remain indistinguishable, the simulator $\mathcal{S}$ needs to build an equivocal commitment $\tilde{\psi}$ without knowing b in the committing phase, where $\tilde{\psi}$ is computational indistinguishable from the real commitment ψ; later in the opening phase, $\mathcal{S}$ obtains b from $\mathcal{F}_{\mathsf{tCOM}}$ and needs to open the previously sent commitment $\tilde{\psi}$ to b. For notation convenience, we write $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2)$ to split the simulator algorithm in two phases: (i) $\mathcal{S}_1$ works in the committing phase, and it needs to output an equivocal commitment $\tilde{\psi}$ without knowing b; (ii) while $\mathcal{S}_2$ works in the opening phase, and upon receiving the message b from $\mathcal{F}_{\mathsf{tCOM}}$, it needs to output the opening message r such that (b, r) correctly opens the previously sent commitment $\tilde{\psi}$.

We first describe the simulation strategy in the committing phase. Recall that, the main advantage of the simulator over the others is that it can obtain illegitimate queries of R^*. More precisely, $\mathcal{S}_1$ can request the illegitimate queries $\mathcal{Q}_{\mathsf{sid}_1}$ from the commitment functionality $\mathcal{F}_{\mathsf{tCOM}}$ who forwards this request to $\mathcal{G}_{\mathsf{oRO}}$. The simulator $\mathcal{S}_1$ also can query $\mathcal{G}_{\mathsf{oRO}}$ just like normal parties. In order to describe the process of querying to $\mathcal{G}_{\mathsf{oRO}}$, we denote by $\mathcal{G}^*_{\mathsf{oRO}}$ the simplified version of the $\mathcal{G}_{\mathsf{oRO}}$, that is, the $\mathcal{G}_{\mathsf{oRO}}$ with only the QUERY interface. We write $\mathcal{S}_1^{\mathcal{G}^*_{\mathsf{oRO}}}$ to denote the event that $\mathcal{S}_1$ has the access to $\mathcal{G}_{\mathsf{oRO}}$ and can continuously query to $\mathcal{G}_{\mathsf{oRO}}$. With above notations, we will write $\mathcal{S}_1^{\mathcal{G}^*_{\mathsf{oRO}}}(\mathsf{sid}_1, \mathcal{Q}_{\mathsf{sid}_1})$ to denote the output (i.e., the equivocal commitment $\tilde{\psi}$ and the state st) produced by $\mathcal{S}_1$ after querying to $\mathcal{G}_{\mathsf{oRO}}$, when running on the the illegitimate queries $\mathcal{Q}_{\mathsf{sid}_1}$ sent by R^*. We note that, $\mathcal{S}_1$ should be able to handle any PPT environment $\mathcal{Z}$. Consider such a case where $\mathcal{Z}$ instructs R^* to delay all its $\mathcal{G}_{\mathsf{oRO}}$ queries until it receives the opening message. In this case, $\mathcal{S}_1$ finds nothing sent by R^* in $\mathcal{Q}_{\mathsf{sid}_1}$, but should still be able to produce the equivocal commitment $\tilde{\psi}$. In other words, the algorithm $(\tilde{\psi}, \mathsf{st}) \leftarrow \mathcal{S}_1^{\mathcal{G}^*_{\mathsf{oRO}}}(\mathsf{sid}_1, \mathcal{Q}_{\mathsf{sid}_1})$ still works when $\mathcal{Q}_{\mathsf{sid}_1} = \emptyset$, where $\emptyset$ is an empty set; otherwise, the environment $\mathcal{Z}$ will find the distinction. We note that, the algorithm $\mathcal{S}_1^{\mathcal{G}^*_{\mathsf{oRO}}}(\mathsf{sid}_1, \emptyset)$, i.e. we replace the $\mathcal{Q}_{\mathsf{sid}_1}$ with the empty set $\emptyset$, can be run by any party, since the algorithm only makes use of the QUERY interface and anyone can query to $\mathcal{G}_{\mathsf{oRO}}$. Now let us turn to the opening phase. Analogously, we can write $r \leftarrow \mathcal{S}_2^{\mathcal{G}^*_{\mathsf{oRO}}}(\mathsf{sid}_1, \tilde{\psi}, \mathsf{st}, b, \emptyset)$ to denote the event where $\mathcal{S}_2$ can still open $\tilde{\psi}$ to the value b and the corresponding opening message r after querying to $\mathcal{G}_{\mathsf{oRO}}$, even if there is noting sent by R^* in the list of the illegitimate queries (i.e. $\mathcal{Q}_{\mathsf{sid}_1} = \emptyset$). We note that, even if we switch to a session with a different SID, both $\mathcal{S}_1^{\mathcal{G}^*_{\mathsf{oRO}}}(\mathsf{sid}_1, \emptyset)$ and $\mathcal{S}_2^{\mathcal{G}^*_{\mathsf{oRO}}}(\mathsf{sid}_1, \tilde{\psi}, \mathsf{st}, b, \emptyset)$ still work as long as the appropriate inputs are provided.

In the following, we show that the existence of the simulator $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2)$ above contradicts the security of Π against static corruptions, by creating a particular environment $\mathcal{Z}'$ which succeeds in distinguishing $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\mathcal{F}_{\mathsf{tCOM}},\mathcal{S}',\mathcal{Z}'}$ from $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\Pi,\mathcal{A}',\mathcal{Z}'}$ after a static corruption operation for any PPT simulator $\mathcal{S}'$. Let us consider the session with SID sid_2. Our $\mathcal{Z}'$ proceeds by corrupting the committer C^* at first, and then choosing a random bit $b \in \{0,1\}$ which it gives as the input to C^*. Next $\mathcal{Z}'$ instructs C^* to run the algorithm $(\tilde{\psi}, \mathsf{st}) \leftarrow \mathcal{S}_1^{\mathcal{G}^*_{\mathsf{oRO}}}(\mathsf{sid}_2, \emptyset)$ and send $\tilde{\psi}$ to R. When R outputs $(\text{RECEIPT}, \mathsf{sid}_2, C, R)$, $\mathcal{Z}'$ instructs C^* to run the algorithm $r \leftarrow \mathcal{S}_2^{\mathcal{G}^*_{\mathsf{oRO}}}(\mathsf{sid}_2, \tilde{\psi}, \mathsf{st}, b, \emptyset)$ and send (b, r) to R. Finally $\mathcal{Z}'$ waits for R to output b'. In the real world, R always outputs $b' = b$. In the ideal world, $\mathcal{S}'$ should determine the committed value b' from $\tilde{\psi}$ in the committing phase. This means that in the ideal world, we must have that $b' = b$ with probability at most $\frac{1}{2}$, since

the entire computation of $\tilde{\psi}$ is totally independent of b. Therefore, $\mathcal{Z}'$ can distinguish between the real world and the ideal world with probability at least $\frac{1}{2}$, contradicting our assumption that Π is GUC-secure. □

4 Feasibility in the GORO Model

In this section, we propose a 3-round (2 rounds for the committing phase and 1 round for the opening phase) GUC-secure commitment protocol in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world, assuming the straight-line extractable NIWH arguments and perfect-hiding non-interactive equivocal commitment schemes exist. Then we instantiate the building blocks using only Minicrypt assumptions in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world. Therefore, our GUC-secure commitment protocol can be constructed via Minicrypt in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world. Since we prove that it is impossible to construct 2-round GUC-secure commitments in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world in Theorem 1, we stress that our construction is round-optimal.

4.1 Our GUC-Secure Commitment Construction

Recall that a GUC-secure commitment protocol requires two main properties: (i) *Equivocality:* when the receiver is corrupted, the simulator should be able to produce equivocal commitments that can open to any value later; (ii) *Extractability:* when the committer is corrupted, the simulator should be able to extract the committed value from the commitment.

The $\mathcal{G}_{\mathsf{oRO}}$ directly provides the desired extractability. Then we have to design a protocol that capture the equivocality. A natural approach is to employ the perfect-hiding non-interactive equivocal commitments. More precisely, we let the receiver generate the commitment key and send it to the committer in the first round; and then let the committer commit to the message using the equivocal commitment scheme. In order to provide extractability, we let the committer query the $\mathcal{G}_{\mathsf{oRO}}$ on the opening message of the commitment message above. Then we require the committer to commit to the answer of the $\mathcal{G}_{\mathsf{oRO}}$ via another instance of the equivocal commitment scheme. The committer sends all the commitment messages in the second round. The opening phase just takes one round, namely, the committer sends all the opening messages.

The only thing left is to provide the simulator with the advantage of getting the equivocation trapdoor over the others. Our solution is to let the receiver execute the straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world which proves the knowledge of the equivocation trapdoor with respect to the commitment key. The receiver is required to send the proof along with the commitment key in the first round. Subsequently, the simulator can invoke the straight-line extractor to obtain the equivocation trapdoor.

We denote committer algorithm as C and receiver algorithm as R. We denote the event where queries $\mathcal{G}_{\mathsf{oRO}}$ on input x and gets the answer y as $y := \mathsf{oRO}(x)$. We assume ideal private and authenticated channels for all communications. Formally, we present our protocol Π_{tCOM} in Figure 8 and prove the security through Theorem 2.

Theorem 2. *Assume Π_{NIWH} is a straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world. Assume Π_{ECom} is an equivocal commitment scheme. Then the protocol Π_{tCOM} described in Figure 8 GUC-realizes the functionality $\mathcal{F}_{\mathsf{tCOM}}$ depicted in Figure 7 in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world against static malicious corruption.*

Proof. We now prove the security of our protocol Π_{tCOM} by showing it is a GUC-secure realization of $\mathcal{F}_{\mathsf{tCOM}}$. We only need to prove that Π_{tCOM} EUC-realizes $\mathcal{F}_{\mathsf{tCOM}}$ with respect to the shared

Protocol Π_{tCOM}

Primitives: Straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world $\Pi_{\mathsf{NIWH}} = \Pi_{\mathsf{NIWH}}.\{\mathsf{Prove}^{\mathcal{G}_{\mathsf{oRO}}}, \mathsf{Verify}^{\mathcal{G}_{\mathsf{oRO}}}, \mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}\}$, non-interactive equivocal commitment $\Pi_{\mathsf{ECom}} = \Pi_{\mathsf{ECom}}.\{\mathsf{KeyGen}, \mathsf{KeyVer}, \mathsf{Commit}, \mathsf{ComVer}, \mathsf{EquCom}, \mathsf{Equiv}\}$.
Inputs: C has a private input $m \in \{0,1\}^\lambda$. R has no input.

Committing Phase: This phase consists of 2 rounds.

- Round 1: R works as follows:
  - Generate the parameters of the commitment by invoking $(\mathsf{ck}, \mathsf{td}) \leftarrow \Pi_{\mathsf{ECom}}.\mathsf{KeyGen}(1^\lambda)$.
  - Compute the straight-line extractable NIWH proof by invoking $\pi \leftarrow \Pi_{\mathsf{NIWH}}.\mathsf{Prove}^{\mathcal{G}_{\mathsf{oRO}}}(\mathsf{ck}, \mathsf{td})$ for proving the knowledge of td. Send (ck, π) to C.
- Round 2: C works as follows:
  - Abort if $\Pi_{\mathsf{NIWH}}.\mathsf{Verify}^{\mathcal{G}_{\mathsf{oRO}}}(\mathsf{ck}, \pi) = 0$.
  - Commit to the message m by invoking $(c_1, d_1) \leftarrow \Pi_{\mathsf{ECom}}.\mathsf{Commit}(\mathsf{ck}, m)$.
  - Compute $h := \mathsf{oRO}(\mathsf{sid}, \text{`}C\text{'}||m||d_1||r)$ where $r \leftarrow \{0,1\}^\lambda$.
  - Commit to the answer h by invoking $(c_2, d_2) \leftarrow \Pi_{\mathsf{ECom}}.\mathsf{Commit}(\mathsf{ck}, h)$. Send (c_1, c_2) to R.

Opening Phase: This phase consists of 1 round.

- Round 3: C sends (m, d_1, d_2, r) to R.
- R computes $h := \mathsf{oRO}(\mathsf{sid}, \text{`}C\text{'}||m||d_1||r)$, and accepts m if and only if $\Pi_{\mathsf{ECom}}.\mathsf{ComVer}(\mathsf{ck}, c_1, d_1) = \Pi_{\mathsf{ECom}}.\mathsf{ComVer}(\mathsf{ck}, c_2, d_2) = 1$ holds.

Figure 8: Protocol Π_{tCOM} in the $\mathcal{G}_{\mathsf{oRO}}$ Hybrid World

functionality $\mathcal{G}_{\mathsf{oRO}}$. Therefore, we describe the workflow of $\mathcal{S}$ in the ideal-world with $\mathcal{F}_{\mathsf{tCOM}}$, and give a proof that the simulation in the ideal-world setting $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\mathcal{F}_{\mathsf{tCOM}},\mathcal{S},\mathcal{Z}}$ is computationally indistinguishable from a real-world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\Pi_{\mathsf{tCOM}},\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{G}_{\mathsf{oRO}}$-constrained environment $\mathcal{Z}$.

Simulating Communication with $\mathcal{Z}$. The simulator $\mathcal{S}$ simply forwards the communication between $\mathcal{A}$ and $\mathcal{Z}$.

Simulating Two Honest Parties. Since we are in the secure channels model, $\mathcal{S}$ simply notifies $\mathcal{A}$ that communications (with messages of appropriate length) have taken place between the committer C and the receiver R.

Simulating the Honest Committer Against the Malicious Receiver. In this case, we suppose that the committer C is honest while the receiver R^* is statically corrupted. Here $\mathcal{S}$ needs to send a simulated commitment without knowing the message m in the committing phase, and open it to the message m in the opening phase. We describe the strategy of $\mathcal{S}$ as follows:

- Commitment Phase: Upon receiving $(\textsc{Receipt}, \mathsf{sid}, C, R)$ from $\mathcal{F}_{\mathsf{tCOM}}$, do
  - Round 1: Wait until the message (ck, π) arrives. Check if $\Pi_{\mathsf{NIWH}}.\mathsf{Verify}^{\mathcal{G}_{\mathsf{oRO}}}(\mathsf{ck}, \pi) = 1$ holds. If not, abort; otherwise, employ the straight-line extractor $\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}(\mathsf{ck}, \pi)$ to compute td s.t. $\Pi_{\mathsf{ECom}}.\mathsf{KeyVer}(\mathsf{ck}, \mathsf{td}) = 1$ (note that, the proof π is verified and the simulator $\mathcal{S}$ is granted the observability of $\mathcal{G}_{\mathsf{oRO}}$, therefore, the simulator $\mathcal{S}$ is able to invoke the extraction algorithm $\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}$). Abort if the straight-line extractor fails.

– Round 2: Generate equivocal commitments $(c_1, \mathsf{st}_1) \leftarrow \Pi_{\mathsf{ECom}}.\mathsf{EquCom}(\mathsf{ck}, \mathsf{td})$ and $(c_2, \mathsf{st}_2) \leftarrow \Pi_{\mathsf{ECom}}.\mathsf{EquCom}(\mathsf{ck}, \mathsf{td})$. Send (c_1, c_2) to R^*.

- Opening Phase: Upon receiving $(\textsc{Decommit}, \mathsf{sid}, C, R, m)$ from $\mathcal{F}_{\mathsf{tCOM}}$, do

  – Round 3: Obtain $d_1 := \Pi_{\mathsf{ECom}}.\mathsf{Equiv}(\mathsf{ck}, \mathsf{td}, c_1, \mathsf{st}_1, m)$. Select a λ-bit random r and compute $h := \mathsf{oRO}(\mathsf{sid}, `C'||m||d_1||r)$. Obtain $d_2 := \Pi_{\mathsf{ECom}}.\mathsf{Equiv}(\mathsf{ck}, \mathsf{td}, c_2, \mathsf{st}_2, h)$. Send (m, d_1, d_2, r) to R^*.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real-world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\Pi_{\mathsf{tCOM}}, \mathcal{A}, \mathcal{Z}}$.

- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ aborts when the straight-line extractor $\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}$ fails to extract a valid trapdoor td such that $\Pi_{\mathsf{ECom}}.\mathsf{KeyVer}(\mathsf{ck}, \mathsf{td}) = 1$.

  Lemma 1. *If Π_{NIWH} is a NIWH argument that has a straight-line extractor $\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}$ with knowledge error $\varepsilon(\lambda)$, then $\mathcal{H}_1$ is indistinguishable from $\mathcal{H}_0$ with adversarial advantage $\varepsilon(\lambda)$.*

  Proof. The probability of $\mathcal{S}$ aborting is equal to the knowledge error $\varepsilon(\lambda)$. Therefore, $\mathcal{H}_1$ is indistinguishable from $\mathcal{H}_0$ with adversarial advantage $\varepsilon(\lambda)$. □

- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ successfully uses the straight-line extractor of Π_{NIWH} to find the valid td, generates the equivocal commitments (c_1, c_2) on dummy strings and equivocate (c_1, c_2) to new openings (d_1, d_2) with respect to (m, h), where $h = \mathsf{oRO}(\mathsf{sid}, `C'||m||d_1||r)$.

  Lemma 2. *If Π_{NIWH} is a NIWH argument that has a straight-line extractor $\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}$ with knowledge error $\varepsilon(\lambda)$, and Π_{ECom} is an equivocal commitment satisfying equivocal property with adversarial advantage $\mathbf{Adv}^{\mathrm{equivocal}}_{\mathcal{A}, \Pi_{\mathsf{ECom}}}(\lambda)$, then $\mathcal{H}_2$ is indistinguishable from $\mathcal{H}_1$ with adversarial advantage $(2 - 2\varepsilon(\lambda)) \cdot \mathbf{Adv}^{\mathrm{equivocal}}_{\mathcal{A}, \Pi_{\mathsf{ECom}}}(\lambda)$.*

  Proof. The probability of $\mathcal{S}$ extracting the valid td such that $\Pi_{\mathsf{ECom}}.\mathsf{KeyVer}(\mathsf{ck}, \mathsf{td}) = 1$ is $1 - \varepsilon(\lambda)$. Since the output of oRO is truly random, h sent by $\mathcal{S}$ is perfect indistinguishable from the one sent by the honest committer. The only thing left is to show the (c_1, c_2, d_1, d_2) are indistinguishable. In this case, we observe that if there is a PPT adversary $\mathcal{A}$ such that $\mathcal{Z}$ can distinguish $\mathcal{H}_1$ from $\mathcal{H}_0$, then we can construct a PPT $\mathcal{B}$ which breaks the equivocal property of the underlying Π_{ECom} scheme. We first focus on the case concerning (c_1, d_1). During the reduction, $\mathcal{B}$ simulates $\mathcal{G}_{\mathsf{oRO}}$ and starts Π_{tCOM} with $\mathcal{A}$ by running $\mathcal{A}$ internally as black-box. First $\mathcal{B}$ waits for $\mathcal{A}$ to send the first round message (ck, π). Then $\mathcal{B}$ invokes the straight-line extractor of the Π_{NIWH} to extract the valid td. After that, $\mathcal{B}$ sends $(\mathsf{ck}, \mathsf{td})$ along with the message m to $\mathcal{C}$. After receiving (c_1, d_1) from $\mathcal{C}$, $\mathcal{B}$ simulates other messages and continues the protocol with $\mathcal{A}$. When $\mathcal{A}$ outputs a bit b, where $b = 0$ indicates (c_1, d_1) is the real commitment-opening pair and $b = 1$ indicates (c_1, d_1) is the equivocated one, $\mathcal{B}$ forwards the same bit b to $\mathcal{C}$. Clearly, $\mathcal{B}$ wins whenever $\mathcal{A}$ wins. The situation concerning (c_2, d_2) is similar. Therefore, we conclude that $\mathcal{H}_3$ is indistinguishable from $\mathcal{H}_2$ with adversarial advantage $(2 - 2\varepsilon(\lambda)) \cdot \mathbf{Adv}^{\mathrm{equivocal}}_{\mathcal{A}, \Pi_{\mathsf{ECom}}}(\lambda)$. □

Hybrid $\mathcal{H}_3$ is identical to the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\mathcal{F}_{\mathsf{tCOM}},\mathcal{S},\mathcal{Z}}$. In conclusion, when receiver is corrupted, the overall distinguishing advantage is at most $\varepsilon(\lambda) + (2 - 2\varepsilon(\lambda)) \cdot \mathbf{Adv}^{\text{equivocal}}_{\mathcal{A},\Pi_{\mathsf{ECom}}}(\lambda)$.

Simulating the Honest Receiver Against the Malicious Committer. In this case, we suppose that the committer C^* is statically corrupted while the receiver R is honest. Here $\mathcal{S}$ needs to extract the committed value from the commitment sent by C^* in the committing phase. We describe the strategy of $\mathcal{S}$ as follows:

- Committing Phase:
  - Round 1: Act as an honest receiver in Round 1 described in Π_{tCOM}.
  - Round 2: Upon receiving (c_1, c_2) from C^*, request the illegitimate queries from $\mathcal{F}_{\mathsf{tCOM}}$. After receiving $\mathcal{Q}_{\mathsf{sid}}$, check if there exists a query of the form $(\mathsf{sid}, \text{`}C\text{'}||m||d_1||r)$ such that $\Pi_{\mathsf{ECom}}.\mathsf{ComVer}(\mathsf{ck}, c_1, d_1) = 1$. If so, set $m' := m$; otherwise, set $m' := 0^\lambda$. Send $(\textsc{Commit}, \mathsf{sid}, C, R, m')$ to $\mathcal{F}_{\mathsf{tCOM}}$ on behalf the dummy committer.
- Opening Phase:
  - Round 3: Act as a honest receiver described in Round 3 in Π_{tCOM}. If the receiver algorithm accepts m^* from C^*, then check if $m^* = m'$. If so, send $(\textsc{Decommit}, \mathsf{sid}, C, R)$ to $\mathcal{F}_{\mathsf{tCOM}}$ on behalf the dummy committer; otherwise, abort.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real-world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\Pi_{\mathsf{tCOM}},\mathcal{A},\mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that in the committing phase, $\mathcal{S}$ obtains $\mathcal{Q}_{\mathsf{sid}}$, determines m' and sends $(\textsc{Commit}, \mathsf{sid}, C, R, m')$ to $\mathcal{F}_{\mathsf{tCOM}}$. The perfect indistinguishability between $\mathcal{H}_0$ and $\mathcal{H}_1$ is trivial since $\mathcal{S}$ does not modify anything.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ aborts in the opening phase when $m^* \neq m'$.

  Lemma 3. *If Π_{NIWH} is a NIWH argument satisfying witness hiding property with adversarial advantage $\mathbf{Adv}^{\text{WH}}_{\mathcal{A},\Pi_{\mathsf{NIWH}}}(\lambda)$, Π_{ECom} is an equivocal commitment scheme satisfying binding property with adversarial advantage $\mathbf{Adv}^{\text{binding}}_{\mathcal{A},\Pi_{\mathsf{ECom}}}(\lambda)$, and $\mathcal{G}_{\mathsf{oRO}}$ is parameterized by the output length $\ell_{\mathsf{out}}(\lambda)$, then $\mathcal{H}_2$ is indistinguishable from $\mathcal{H}_1$ with advantage $\mathbf{Adv}^{\text{WH}}_{\mathcal{A},\Pi_{\mathsf{NIWH}}}(\lambda) + \mathbf{Adv}^{\text{binding}}_{\mathcal{A},\Pi_{\mathsf{ECom}}}(\lambda) + 2^{-\ell_{\mathsf{out}}(\lambda)}$.*

  Proof. Here $\mathcal{S}$ aborts when $m^* \neq m'$. This would happen when (i) C^* does not query $\mathcal{G}_{\mathsf{oRO}}$ on input $(\mathsf{sid}, \text{`}C\text{'}||m||d_1||r)$, but guesses the output of $\mathcal{G}_{\mathsf{oRO}}$ precisely; (ii) C^* is able to open the commitment sent earlier to any value. It is easy to see that the former case would happen at at a negligible probability, namely $2^{-\ell_{\mathsf{out}}(\lambda)}$.

  Now let us focus on the latter case. The witness hiding property guarantees that the corrupted committer cannot obtain the td. Therefore, we observe that if there is a PPT adversary $\mathcal{A}$ such that $\mathcal{Z}$ can distinguish $\mathcal{H}_2$ from $\mathcal{H}_1$, then we can construct a PPT $\mathcal{B}$ which breaks the binding property of the underlying Π_{ECom}. During the reduction, $\mathcal{B}$ simulates $\mathcal{G}_{\mathsf{oRO}}$ and starts Π_{tCOM} with $\mathcal{A}$ by running $\mathcal{A}$ internally as black-box. First $\mathcal{B}$ interacts with the binding game challenger $\mathcal{C}$, and receives ck from $\mathcal{C}$. Then $\mathcal{B}$ simulates the NIWH proof π and sends (ck, π) to $\mathcal{A}$. When $\mathcal{A}$ makes $\mathcal{S}$ aborts, i.e., $\mathcal{A}$ queries $\mathcal{G}_{\mathsf{oRO}}$ on input $m||d_1||r$, but later sends another valid opening d_1', $\mathcal{B}$ sends c_1, d_1, d_1' to $\mathcal{C}$. It is easy to see that $\mathcal{B}$ wins whenever $\mathcal{A}$ wins. We conclude that $\mathcal{H}_2$ is indistinguishable from $\mathcal{H}_1$ with adversarial advantage $\mathbf{Adv}^{\text{WH}}_{\mathcal{A},\Pi_{\mathsf{NIWH}}}(\lambda) + \mathbf{Adv}^{\text{binding}}_{\mathcal{A},\Pi_{\mathsf{ECom}}}(\lambda) + 2^{-\ell_{\mathsf{out}}(\lambda)}$. □

Hybrid $\mathcal{H}_3$ is identical to the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{oRO}}}_{\mathcal{F}_{\mathsf{tCOM}},\mathcal{S},\mathcal{Z}}$. In conclusion, when committer is statically corrupted, the overall distinguishing advantage is at most $\mathbf{Adv}^{\mathrm{WH}}_{\mathcal{A},\Pi_{\mathsf{NIWH}}}(\lambda) + \mathbf{Adv}^{\mathrm{binding}}_{\mathcal{A},\Pi_{\mathsf{ECom}}}(\lambda) + 2^{-\ell_{\mathsf{out}}(\lambda)}$.

In conclusion, the protocol Π_{tCOM} GUC-realizes $\mathcal{F}_{\mathsf{tCOM}}$ in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world against static malicious corruption with advantage $\max\{\varepsilon(\lambda) + (2 - 2\varepsilon(\lambda)) \cdot \mathbf{Adv}^{\mathrm{equivocal}}_{\mathcal{A},\Pi_{\mathsf{ECom}}}(\lambda), \mathbf{Adv}^{\mathrm{WH}}_{\mathcal{A},\Pi_{\mathsf{NIWH}}}(\lambda) + \mathbf{Adv}^{\mathrm{binding}}_{\mathcal{A},\Pi_{\mathsf{ECom}}}(\lambda) + 2^{-\ell_{\mathsf{out}}(\lambda)}\}$. This completes the proof. □

4.2 Instantiation of the Building Blocks

There are two building blocks, i.e. straight-line extractable NIWH arguments and perfect-hiding non-interactive equivocal commitment schemes, in our construction. In this section, we show how to instantiate them using only Minicrypt assumptions in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world. We start by constructing a SHVZK protocol, since it is needed in both building blocks.

4.2.1 SHVZK Protocols from "MPC-in-the-Head"

In this section, we aim to construct a SHVZK protocol using only Minicrypt assumptions in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world. Our starting point is the "MPC-in-the-head" paradigm proposed by Ishai *et al.* [IKOS07] which is introduced in Section 2.8.

Note that our construction requires an SHVZK protocol with 2-special soundness (, which we will explain the necessarity later in Section 4.2.2, below); unfortunately, to the best of our knowledge, none of the followups [GMO16, CDG+17, AHIV17, KKW18, dOT21] since the original work of [IKOS07], can lead to a 2-special sound protocol based on only Minicrypt assumptions. Therefore, we need to design a new technique approach that transforms a MPC protocol into a SHVZK protocol with 2-special soundness.

Our starting point: [KKW18]. We start with the 5-round SHVZK protocol proposed by Katz *et al.* in [KKW18] which is based on only Minicrypt assumptions. In the high-level description, Katz *et al.* employed the $(n-1)$-private n-player MPC protocol in the preprocessing model and let the verifier provide its challenges in *two* phases: one for checking the correctness of the opened preprocessing execution, and the other for checking the consistency of the opened views. Roughly speaking, the 5-round protocol of Katz *et al.* works as follows:

- Round 1: The prover simulates m independent executions of the preprocessing phase, and commits to the states of the parties which can be obtained at the end of the preprocessing phase.
- Round 2: The verifier samples an uniform random challenge $c \in [m]$ and asks the prover to open the views of all the executions of the preprocessing phase except the c-th one.
- Round 3: The prover opens the states of all parties for each challenged execution of preprocessing phase. Beside that, the prover simulates the execution of Π_{MPC} that realizes $f_{\mathcal{R}}$ using the remaining unopened execution of the preprocessing phase. The prover then commits to each view of the parties.
- Round 4: The verifier samples an uniform random challenge $p \in [n]$ and asks the prover to open all the views of the parties except the p-th one.

- Round 5: The prover reveals the state of each challenged party following the preprocessing phase as well as its view in the execution of Π_{MPC}. The verifier checks that the opened views are consistent with each other and with an honest execution of Π_{MPC} (using the state from the preprocessing phase) that yields the output 1.

In [KKW18], Katz *et al.* compressed the above 5-round protocol into a 3-round one by the following approach: (i) let the prover simulate the execution of Π_{MPC} for every emulation of the preprocessing phase and commit to all the resulting views as well as the states; (ii) let the verifier send its challenge (c, p), and asks the prover to open all the states except the c-th one of the preprocessing phase as well as all the views except the p-th one from the unopened preprocessing phase.

Let Π_{MPC} be the n-party MPC protocol which realizes f with $(n-1)$-privacy in the preprocessing model, where $f(x, w_1, \cdots, w_n) = \mathcal{R}_{\mathcal{L}}(x, w_1 \oplus \cdots \oplus w_n)$. Let $\mathsf{Preprocess}$ be the preprocessing algorithm that takes a λ-bit random string seed as input, and outputs the states $\{\mathsf{state}\}_{i\in[n]}$ which are used for the computation later (cf. [KKW18] for details). We use the $\mathcal{G}_{\mathsf{oRO}}$ to instantiate the statically binding commitment (i.e., to commit msg with random coin r, we use the answer of the $\mathcal{G}_{\mathsf{oRO}}$ on input (msg, r) as the commitment and reveal (msg, r) as the opening). We denote the event where queries $\mathcal{G}_{\mathsf{oRO}}$ on input x and gets the answer y as $y := \mathsf{oRO}(x)$ in the context. We denote by m the number of the executions of the preprocessing phase. Formally, we recall the 3-round SHVZK protocol $\Pi^{\mathsf{KKW}}_{\mathsf{SHVZK}}$ from [KKW18] in Figure 9. We emphasize that the protocol $\Pi^{\mathsf{KKW}}_{\mathsf{SHVZK}}$ cannot be 2-special sound, and we argue this through Proposition 1.

Proposition 1. *Assume the n-party MPC protocol Π_{MPC} is $(n-1)$-private in the preprocessing model. Let m be the number of executions of preprocessing phase, where $m \geq 2$. The 3-round SHVZK protocol $\Pi^{\mathsf{KKW}}_{\mathsf{SHVZK}}$ depicted in Figure 9:*

- *cannot achieve k-special soundness, for $k \leq m$.*
- *satisfies k-special soundness, for $k \geq m+1$.*

Proof. In order to prove the proposition above, we have to show that (i) given any k distinct valid transcripts, where $k \geq m+1$, there exists a PPT algorithm that can efficiently recover the witness; (ii) there always exists a PPT adversary $\mathcal{A}$ that can produce k distinct valid transcripts without knowing the witness, where $k \leq m$.

We first discuss the former case. Recall that the challenge of the protocol is (c, p), and the domain of the first challenge c is $[m]$. Therefore, given k distinct transcripts where $k \geq m+1$, there must exist a $c^* \in [m]$ and $p \neq p'$ such that (c^*, p) and (c^*, p') are the challenges that corresponds to two of the given transcripts. For transcript that corresponds to (c^*, p), it reveals $\{\mathsf{state}_{c^*,j}, \mathsf{view}_{c^*,j}(x, w_j)\}_{j\in[n]\setminus\{p\}}$; and for transcript that corresponds to (c^*, p'), it reveals $\{\mathsf{state}_{c^*,j}, \mathsf{view}_{c^*,j}(x, w_j)\}_{j\in[n]\setminus\{p'\}}$. Since $p \neq p'$, we obtain $\{\mathsf{state}_{c^*,j}, \mathsf{view}_{c^*,j}(x, w_j)\}_{j\in[n]}$. Then we yield the private input w_j from the $\mathsf{state}_{c^*,j}, \mathsf{view}_{c^*,j}(x, w_j)$, and finally computes $w := w_1 \oplus \cdots \oplus w_n$. Therefore, there exists a PPT algorithm that can efficiently recover the witness w when given $m+1$ distinct valid transcripts.

We then discuss the latter case. For $k \leq m$, we can construct a PPT adversary $\mathcal{A}$ that produces k distinct accepting transcripts without knowing the witness w as follows:

- Sample uniformly random $p \in [n]$.
- For $i \in [k]$:
  - Sample λ-bit randomness seed_i and generate $\{\mathsf{state}_{i,j}\}_{j\in[n]} \leftarrow \mathsf{Preprocess}(\mathsf{seed}_i)$.

Protocol $\Pi^{\mathsf{KKW}}_{\mathsf{SHVZK}}$

Primitives: n-party MPC protocol Π_{MPC} which realizes f with $(n-1)$-privacy in the preprocessing model, where $f(x, w_1, \ldots, w_n) = \mathcal{R}_{\mathcal{L}}(x, w_1 \oplus \cdots \oplus w_n)$.
Inputs: P, V have a public input x and an NP relation $\mathcal{R}_{\mathcal{L}}$. P has a private input w s.t. $\mathcal{R}_{\mathcal{L}}(x, w) = 1$.

Protocol:

- $\mathsf{Move1}(x, w; r)$:
  - For $i \in [m]$:
    - Derive λ-bit random seed_i from randomness r and generate $\{\mathsf{state}_{i,j}\}_{j\in[n]} \leftarrow \mathsf{Preprocess}(\mathsf{seed}_i)$.
    - Simulate the execution of Π_{MPC} using (x, w) and the states $\{\mathsf{state}_{i,j}\}_{j\in[n]}$, and output the views of the parties $\{\mathsf{view}_{i,j}(x, w_j)\}_{j\in[n]}$.
    - For $j \in [n]$: select random $r_{i,j}, \widetilde{r}_{i,j} \leftarrow \{0,1\}^\lambda$, and compute the state-commitments $\mathsf{com}_{i,j} := \mathsf{oRO}(\mathsf{sid}, \mathsf{state}_{i,j} || r_{i,j})$ and the view-commitments $\widetilde{\mathsf{com}}_{i,j} := \mathsf{oRO}(\mathsf{sid}, \mathsf{view}_{i,j}(x, w_j) || \widetilde{r}_{i,j})$.
  - Send $a := (\{\mathsf{com}_{i,j}, \widetilde{\mathsf{com}}_{i,j}\}_{i\in[m],j\in[n]})$.
- $\mathsf{Move2}(1^\lambda)$: Send $e := (c, p)$, where c is uniformly random in $[m]$ and p is uniformly random in $[n]$.
- $\mathsf{Move3}(x, w, e; r)$: Send $z := (\{\mathsf{state}_{i,j}, r_{i,j}\}_{i\in[m]\setminus\{c\},j\in[n]}, \{\mathsf{state}_{c,j}, \mathsf{view}_{c,j}(x, w_j), \tilde{r}_{c,j}\}_{j\in[n]\setminus\{p\}})$.
- $\mathsf{Verify}(x, a, e, z)$: Output 1 if and only if the following checks pass:
  - Check the commitments are opened correctly:
    - For $i \in [m] \setminus \{c\}, j \in [n]$: check $\mathsf{com}_{i,j} = \mathsf{oRO}(\mathsf{sid}, \mathsf{state}_{i,j} || r_{i,j})$ holds.
    - For $j \in [n] \setminus \{p\}$: check $\mathsf{com}_{c,j} = \mathsf{oRO}(\mathsf{sid}, \mathsf{state}_{c,j} || r_{c,j})$ and $\widetilde{\mathsf{com}}_{c,j} = \mathsf{oRO}(\mathsf{sid}, \mathsf{view}_{c,j}(x, w_j) || \tilde{r}_{i,j})$ hold.
  - Check the correctness of the executions of the preprocessing phase:
    - For $i \in [m] \setminus \{c\}$: check $\{\mathsf{state}_{i,j}\}_{j\in[n]}$ are well-formed.
  - Check the consistency between the opened views:
    - For $j \in [n] \setminus \{p\}$: check $\mathsf{view}_{c,j}(x, w_j)$ correctly follows from the $\mathsf{state}_{c,j}$ and $\mathsf{view}_{c,j}(x, w_j)$ yields output 1.
    - Check $\{\mathsf{view}_{c,j}(x, w_j)\}_{\in[n]\setminus\{p\}}$ are consistent with each other.

Figure 9: Protocol $\Pi^{\mathsf{KKW}}_{\mathsf{SHVZK}}$ from [KKW18]

- Run the simulator algorithm of Π_{MPC} using x, p and the states generated by the i-th preprocessing phase (i.e., $\{\mathsf{state}_{i,j}\}_{j\in[n]}$), and output the simulated views $\{\mathsf{view}_{i,j}\}_{\in[n]\setminus\{p\}}$. Sample a random $\mathsf{view}_{i,p}$ of appropriate length.
- For $j \in [n]$: select λ-bit randomness $r_{i,j}, \widetilde{r}_{i,j}$, and compute $\mathsf{com}_{i,j} := \mathsf{oRO}(\mathsf{sid}, \mathsf{state}_{i,j}||r_{i,j})$ and $\widetilde{\mathsf{com}}_{i,j} := \mathsf{oRO}(\mathsf{sid}, \mathsf{view}_{i,j}(x, w_j)||\widetilde{r}_{i,j})$.
- Set $e_i := (i, p)$ and $z_i := (\{\mathsf{state}_{k,j}, r_{k,j}\}_{k\neq i, j\in[n]}, \{\mathsf{state}_{i,j}, \mathsf{view}_{i,j}(x, w_j), \tilde{r}_{i,j}\}_{j\in[n]\setminus\{p\}})$.

- Set $a := (\{\mathsf{com}_{i,j}, \widetilde{\mathsf{com}}_{i,j}\}_{i\in[m],j\in[n]})$, and output $\{(a, e_i, z_i)\}_{i\in[m]}$.

It is easy to see that (a, e_i, z_i) is an accepting transcript for every $i \in [k]$, and $\{e_i\}_{i\in[k]}$ are distinct with each other where $e_i = (i, p)$. Therefore, there exists a PPT adversary $\mathcal{A}$ that can produce k distinct valid transcripts without knowing w, where $k \leq m$. In other words, given only k distinct valid transcripts where $k \leq m$, it is not guaranteed that we can recover the witness from them.

In conclusion, we prove that $\Pi^{\mathsf{KKW}}_{\mathsf{SHVZK}}$: (i) cannot achieve k-special soundness, for $k \leq m$; (ii) satisfies k-special soundness, for $k \geq m + 1$. □

Remark 2. *Katz* et al. *proposed a generalized 3-round SHVZK protocol in [KKW18] for better efficiency. More precisely, they let the verifier select $m - \tau$ (instead of $m - 1$) of the executions of the preprocessing phase to check, where τ is a parameter and $1 \leq \tau \leq m - 1$; the remaining τ executions of the preprocessing phase are used to run τ (instead of one) instances of the MPC protocol Π_{MPC}; each of the executions of Π_{MPC} is verified by revealing the view of all-but-one party as before. We note that, this generalized SHVZK protocol cannot achieve k-special soundness, where $k \leq \binom{m-1}{\tau} + 1$; but it satisfies k-special soundness, where $k \geq \binom{m-1}{\tau} + 2$. The proof is analogously to the Proposition 1, and we omit the proof here.*

Our protocol construction. Our key observation is that we can compress the above 5-round protocol into a 3-round one by applying the Fiat-Shamir transformation [FS87] to replace Round 2. Therefore, Round 1 and Round 3 can be merged, and we obtain a 3-round protocol with 2-special soundness. We can regard the first round of the resulting 3-round protocol as a "non-interactive proof" that proves the correctness of the execution of the preprocessing phase, but its soundness error is not negligible (i.e., $\frac{1}{m}$, where m is the number of the executions of preprocessing phase). This issue can be addressed by applying parallel repetition. Compared with the approach of [KKW18], our approach needs additional RO assumptions but it is an SHVZK protocol with 2-special sound.

Let Π_{MPC} be the n-party MPC protocol which realizes f with $(n-1)$-privacy in the preprocessing model, where $f(x, w_1, \cdots, w_n) = \mathcal{R}_{\mathcal{L}}(x, w_1 \oplus \cdots \oplus w_n)$. Let $\mathsf{Preprocess}$ be the preprocessing algorithm that takes a λ-bit random string seed as input, and outputs the states $\{\mathsf{state}\}_{i\in[n]}$ which are used for the computation later (cf. [KKW18] for details). We denote the event where queries $\mathcal{G}_{\mathsf{oRO}_i}$ on input x and gets the answer y as $y := \mathsf{oRO}_i(x)$ for $i \in \{1, 2\}$ in the context, where $\mathsf{oRO}_1 : \{0,1\}^{\ell_{\mathsf{in}}(\lambda)} \to \{0,1\}^{\ell}$ and $\mathsf{oRO}_2 : \{0,1\}^{\ell_{\mathsf{in}}(\lambda)} \to (\mathbb{Z}^+_{m+1})^{\lambda}$. We denote by m the number of the executions of the preprocessing phase. Formally, we present our protocol Π_{SHVZK} in Figure 10 and prove the security through Theorem 3.

Theorem 3. *Assume Π_{MPC} is a secure n-party protocol that realizes $f_{\mathcal{R}}$ with perfect $(n-1)$-privacy, where $f(x, w_1, \cdots, w_n) = \mathcal{R}_{\mathcal{L}}(x, w_1 \oplus \cdots \oplus w_n)$. Then the protocol Π_{SHVZK} depicted in Figure 10 is a SHVZK protocol that is perfect complete, 2-special sound, perfect SHVZK.*

Proof. Perfect Completeness. It is straightforward.

Perfect SHVZK. In order to show perfect SHVZK, We have to present the simulation strategy first. Given the statement x, the challenge $e = (p_1, \ldots, p_\lambda)$ and the random string r, the simulator $\mathsf{Sim}(x, w; r)$ works as follows:

Protocol Π_{SHVZK}

Primitives: n-party MPC protocol Π_{MPC} which realizes f with $(n-1)$-privacy in the preprocessing model, where $f(x, w_1, \ldots, w_n) = \mathcal{R}_{\mathcal{L}}(x, w_1 \oplus \cdots \oplus w_n)$.
Random Oracles: $\mathsf{oRO}_1 : \{0,1\}^{\ell_{\mathsf{in}}(\lambda)} \to \{0,1\}^{\ell}$ and $\mathsf{oRO}_2 : \{0,1\}^{\ell_{\mathsf{in}}(\lambda)} \to (\mathbb{Z}_{m+1}^{+})^{\lambda}$
Inputs: P, V have a common x and an NP relation $\mathcal{R}_{\mathcal{L}}$. P has a private w s.t. $\mathcal{R}_{\mathcal{L}}(x, w) = 1$.

Protocol:

- $\mathsf{Move1}(x, w; r)$:
  - For $i \in [\lambda], j \in [m]$:
    - Derive λ-bit random $\mathsf{seed}_{i,j}$ from randomness r and generate $\{\mathsf{state}_{i,j,k}\}_{k\in[n]} \leftarrow \mathsf{Preprocess}(\mathsf{seed}_{i,j})$.
    - For $k \in [n]$: select $r_{i,j,k} \leftarrow \{0,1\}^{\lambda}$ and commit to the states, i.e. compute state-commitments $\mathsf{com}_{i,j,k} := \mathsf{oRO}_1(\mathsf{sid}, \mathsf{state}_{i,j,k} || r_{i,j,k})$.
  - Compute $(c_1, \ldots, c_\lambda) := \mathsf{oRO}_2(\mathsf{sid}, \{\mathsf{com}_{i,j,k}\}_{i\in[\lambda],j\in[m],k\in[n]})$, where $c_i \in [m]$.
  - For $i \in [\lambda]$:
    - Simulate the execution of Π_{MPC} using (x, w) and the states generated by the c_i-th preprocessing phase (i.e., $\{\mathsf{state}_{i,c_i,k}\}_{k\in[n]}$), and output the views of the parties $\{\mathsf{view}_{i,k}(x, w_k)\}_{k\in[n]}$.
    - For $k \in [n]$: select $\tilde{r}_{i,k} \leftarrow \{0,1\}^{\lambda}$ and commit to the view of each party, i.e. compute view-commitments $\widetilde{\mathsf{com}}_{i,k} := \mathsf{oRO}_1(\mathsf{sid}, \mathsf{view}_{i,k}(x, w_k) || \tilde{r}_{i,k})$.
  - Send $a := (\{\mathsf{com}_{i,j,k}, \widetilde{\mathsf{com}}_{i,k}\}_{i\in[\lambda],j\in[m],k\in[n]}, \{\mathsf{state}_{i,j,k}, r_{i,j,k}\}_{i\in[\lambda],j\in[m]\setminus\{c_i\},k\in[n]})$
- $\mathsf{Move2}(1^{\lambda})$: Send $e := (p_1, \ldots, p_\lambda)$, where $p_i \in [n]$ and p_i is uniformly random.
- $\mathsf{Move3}(x, w, e; r)$: Send $z := (\{\mathsf{view}_{i,k}(x, w_k), \tilde{r}_{i,k}, \mathsf{state}_{i,c_i,k}, r_{i,c_i,k}\}_{i\in[\lambda],k\in[n]\setminus\{p_i\}})$.
- $\mathsf{Verify}(x, a, e, z)$: Output 1 if and only if the following checks pass:
  - Check the commitments are opened correctly:
    - For $i \in [\lambda], j \in [m] \setminus \{c_i\}, k \in [n]$: check $\mathsf{com}_{i,j,k} = \mathsf{oRO}_1(\mathsf{sid}, \mathsf{state}_{i,j,k} || r_{i,j,k})$ holds.
    - For $i \in [\lambda], k \in [n] \setminus \{p_i\}$: check $\mathsf{com}_{i,c_i,k} = \mathsf{oRO}_1(\mathsf{sid}, \mathsf{state}_{i,c_i,k} || r_{i,c_i,k})$ and $\widetilde{\mathsf{com}}_{i,k} = \mathsf{oRO}_1(\mathsf{sid}, \mathsf{view}_{i,k}(x, w_k) || \tilde{r}_{i,k})$ hold.
  - Check the correctness of the executions of the preprocessing phase:
    - Compute $(c_1, \ldots, c_\lambda) := \mathsf{oRO}_2(\mathsf{sid}, \{\mathsf{com}_{i,j,k}\}_{i\in[\lambda],j\in[m],k\in[n]})$.
    - For $i \in [\lambda], j \in [m] \setminus \{c_i\}$: check $\{\mathsf{state}_{i,j,k}\}_{k\in[n]}$ are well-formed.
  - Check the consistency between the opened views:
    - For $i \in [\lambda], k \in [n] \setminus \{p_i\}$: check $\mathsf{view}_{i,k}(x, w_k)$ follows from the $\mathsf{state}_{i,c_i,k}$ correctly and $\mathsf{view}_{i,k}(x, w_k)$ yields output 1.
    - For $i \in [\lambda]$: check $\{\mathsf{view}_{i,k}(x, w_k)\}_{k\in[n]\setminus\{p_i\}}$ are consistent with each other.

Figure 10: Protocol Π_{SHVZK} in the $\mathcal{G}_{\mathsf{oRO}}$ Hybrid World

- For $i \in [\lambda], j \in [m]$:
  - Derive λ-bit random $\mathsf{seed}_{i,j}$ from r and generate $\{\mathsf{state}_{i,j,k}\}_{k\in[n]} \leftarrow \mathsf{Preprocess}(\mathsf{seed}_{i,j})$.
  - For $k \in [n]$: select $r_{i,j,k} \leftarrow \{0,1\}^\lambda$ and commit to states $\mathsf{com}_{i,j,k} := \mathsf{oRO}_1(\mathsf{sid}, \mathsf{state}_{i,j,k} || r_{i,j,k})$.
- Compute $(c_1, \ldots, c_\lambda) := \mathsf{oRO}_2(\mathsf{sid}, \{\mathsf{com}_{i,j,k}\}_{i\in[\lambda],j\in[m],k\in[n]})$, where $c_i \in [m]$ for $i \in [\lambda]$.
- For $i \in [\lambda]$:
  - Run the simulator algorithm of Π_{MPC} using x, p_i and the states generated by the c_i-th preprocessing phase (i.e., $\{\mathsf{state}_{i,c_i,k}\}_{k\in[n]}$), and output the simulated views $\{\mathsf{view}_{i,k}\}_{k\in[n]\setminus\{p_i\}}$. Sample a random view_{i,p_i} of appropriate length.
  - For $k \in [n]$: select $\tilde{r}_{i,k} \leftarrow \{0,1\}^\lambda$ and commit to the view of each party $\widetilde{\mathsf{com}}_{i,k} := \mathsf{oRO}_1(\mathsf{sid}, \mathsf{view}_{i,k} || \tilde{r}_{i,k})$.
- Output $a := (\{\mathsf{com}_{i,j,k}, \widetilde{\mathsf{com}}_{i,k}\}_{i\in[\lambda],j\in[m],k\in[n]}, \{\mathsf{state}_{i,j,k}, r_{i,j,k}\}_{i\in[\lambda],j\in[m]\setminus\{c_i\},k\in[n]})$ and $z := (\{\mathsf{view}_{i,k}, \tilde{r}_{i,k}, \mathsf{state}_{i,c_i,k}, r_{i,c_i,k}\}_{i\in[\lambda],k\in[n]\setminus\{p_i\}})$.

Now the only thing left is to show the indistinguishability. Since the simulator emulates the executions of the preprocessing phase honestly, the real states of the parties and the simulated ones are perfectly indistinguishable. Now we turn to the views of the parties: given $e = (p_1, \ldots, p_\lambda)$ ahead, by perfect $(n-1)$-privacy of Π_{MPC}, the real opened views of the parties and simulated ones are perfectly indistinguishable. Thus, the real z and simulated z' are perfectly indistinguishable. Due to the unpredictability of the output of $\mathcal{G}_{\mathsf{oRO}}$, the real a and simulated a' are perfectly indistinguishable. Therefore, we prove that our protocol satisfies perfect SHVZK property.

2-Special Soundness. Due to the unpredictability of the random oracle, any PPT adversary can cheat in proving the correctness of the preprocessing phase without being detected with only $m^{-\lambda}$ probability, which is only negligible. In other words, given a in an accepting transcript, the preprocessing phase must be executed correctly with overwhelming probability. Fixing such $a = (\{\mathsf{com}_{i,j,k}, \widetilde{\mathsf{com}}_{i,k}\}_{i\in[\lambda],j\in[m],k\in[n]}, \{\mathsf{state}_{i,j,k}, r_{i,j,k}\}_{i\in[\lambda],j\in[m]\setminus\{c_i\},k\in[n]})$, for any two distinct accepting transcripts $(a, e, z), (a, e', z')$ where $e \neq e'$ and $e = (p_1, \ldots, p_\lambda), e' = (p'_1, \ldots, p'_\lambda)$, we can find a $i \in [\lambda]$ such that $p_i \neq p'_i$. Thus, we can obtain all the committed views $\{\mathsf{view}_{i,k}(x, w_k), \mathsf{state}_{i,c_i,k}\}_{k\in[n]}$ given these transcripts. Then we can yield w_k from $\mathsf{view}_i(x, w_k)$ and $\mathsf{state}_{i,c_i,k}$, and compute $w := w_1 \oplus \cdots \oplus w_n$. Since all the views and states has been verified due to the definition of the special soundness, the extracted witness w must satisfy $(x, w) \in \mathcal{R}_\mathcal{L}$. Therefore, we prove that our protocol is 2-special sound. □

4.2.2 Perfect-Hiding Non-Interactive Equivocal Commitment

Given a SHVZK protocol, we can obtain a perfect-hiding non-interactive equivocal commitment. The intuition is as follows. Let $\mathcal{R}_\mathcal{L}$ be a hard NP relation. The receiver selects $(x, w) \in \mathcal{R}_\mathcal{L}$, and sets x as the commitment key and w as the equivocation trapdoor. The message m is used as the challenge on which to run the simulator for the SHVZK protocol with respect to x, producing the prover's first flow a and the response z. The first flow a is used as the commitment. The message m and response z are used as the opening. Equivocation is achieved by using the knowledge of w to execute the honest prover algorithm instead of the simulator algorithm. Similar ideas can be found in [Dam02, MY04].

Let g be a one-way function. Formally, we present our non-interactive equivocal commitment in Figure 11 and prove the security through Theorem 4. The proof of computational binding relies

on the 2-special soundness, and this explains the reason why 2-special soundness is necessary in Section 4.2.1. We instantiate the NP relation with one-way function, i.e. $\mathcal{R}_1 = \{(y, \mathsf{seed}) \mid y = g(\mathsf{seed})\}$ where (y, seed) is the statement-witness pair and g is a one-way function. If we use our SHVZK protocol Π_{SHVZK} depicted in Figure 10 as the building block, then we can obtain a perfect hiding non-interactive equivocal commitment scheme via only Minicrypt assumptions in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world.

Scheme Π_{ECom}

Primitives: SHVZK protocol $\Pi_{\mathsf{SHVZK}} = \Pi_{\mathsf{SHVZK}}.\{\mathsf{Move1}, \mathsf{Move2}, \mathsf{Move3}, \mathsf{Verify}, \mathsf{Sim}\}$ and one-way function g.

- $\mathsf{KeyGen}(1^\lambda)$: Select a random string $\mathsf{seed} \leftarrow \{0,1\}^\lambda$, compute $y := g(\mathsf{seed})$, and output $\mathsf{ck} := y, \mathsf{td} := \mathsf{seed}$.
- $\mathsf{KeyVer}(\mathsf{ck}, \mathsf{td})$: Check if $\mathsf{ck} = g(\mathsf{td})$ holds. If so, output 1; otherwise, output 0.
- $\mathsf{Commit}(\mathsf{ck}, m)$: Select a random string $r \leftarrow \{0,1\}^\lambda$, invoke $(a, z) := \Pi_{\mathsf{SHVZK}}.\mathsf{Sim}(\mathsf{ck}, m; r)$, and output $c := a, d := (m, z)$.
- $\mathsf{ComVer}(\mathsf{ck}, c, d)$: Check if $\Pi_{\mathsf{SHVZK}}.\mathsf{Verify}(\mathsf{ck}, c, m, z) = 1$ holds. If so, output 1; otherwise, output 0.
- $\mathsf{EquCom}(\mathsf{ck}, \mathsf{td})$: Select a random string $s \leftarrow \{0,1\}^\lambda$, invoke $\tilde{a} := \Pi_{\mathsf{SHVZK}}.\mathsf{Move1}(\mathsf{ck}, \mathsf{td}; s)$, and output $\tilde{c} := \tilde{a}, \mathsf{st} := s$.
- $\mathsf{Equiv}(\mathsf{ck}, \mathsf{td}, \tilde{c}, \mathsf{st}, \tilde{m})$: Invoke $\tilde{z} := \Pi_{\mathsf{SHVZK}}.\mathsf{Move3}(\mathsf{ck}, \mathsf{td}, \tilde{m}; \mathsf{st})$, and output $\tilde{d} := \tilde{z}$.

Figure 11: Scheme Π_{ECom} Based on One-Way Function

Theorem 4. *Assume Π_{SHVZK} is a SHVZK protocol that is 2-special sound and perfect SHVZK. Assume g is a one-way function that is hard to invert. Then Π_{ECom} depicted in Figure 11 is an equivocal commitment that satisfies perfect correctness, perfect hiding, computational binding and perfect equivocation.*

Proof. Perfect Correctness. It is straightforward.

Perfect Hiding. Note that, the first move message of the honest prover algorithm (i.e., the real a, where $a \leftarrow \Pi_{\mathsf{SHVZK}}.\mathsf{Move1}(x, w)$) is totally independent of the challenge m. By perfect SHVZK property of Π_{SHVZK}, the simulated first move message (i.e., the simulated $\tilde{a}$, where $(\tilde{a}, \tilde{z}) \leftarrow \Pi_{\mathsf{SHVZK}}.\mathsf{Sim}(x, m)$) should also be uncorrelated to m. Since $\tilde{a}$ is the commitment to the message m, any computationally unbounded adversary $\mathcal{A}$ cannot learn m from $\tilde{a}$. Therefore, we prove that our construction is perfect hiding.

Computational Binding. We proceed by contradiction. Assume there exists a PPT adversary $\mathcal{A}$ that breaks the computational binding property of the Π_{ECom} with non-negligible probability, then we are able to build a reduction $\mathcal{B}$ which violates the Hard to Invert (HI) property of the underlying g. First $\mathcal{B}$ interacts with HI game challenger $\mathcal{C}$. After receiving y from $\mathcal{C}$, $\mathcal{C}$ forwards y to $\mathcal{A}$. Then $\mathcal{B}$ waits for $\mathcal{A}$ to send $c := a, d := (m_1, z_1), d' := (m_2, z_2)$, where $m_1 \neq m_2$. Due to the 2-special soundness, $\mathcal{B}$ is able to extract the witness seed such that $y = g(\mathsf{seed})$, sends seed to $\mathcal{C}$ and wins the game. Therefore, we prove that our construction is computational binding.

Perfect Equivocation. In order to prove perfect equivocal property, we have to show that the real commitment-opening pair $(c := a, d := (m, z))$ is perfectly indistinguishable from the equivocated commitment-opening pair $(\tilde{c} := \tilde{a}, \tilde{d} := (m, \tilde{z}))$. By perfect SHVZK property of Π_{SHVZK}, (a, z) is perfectly indistinguishable from $(\tilde{a}, \tilde{z})$. Therefore, we prove that our construction is perfect equivocal. □

4.2.3 Straight-Line Extractable NIWH Argument

We construct the straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world using the technique described in [Pas03]. We here describe the high-level description and the details can be found in Appendix B. Given a SHVZK protocol with 2-special soundness, we let the prover execute the honest prover algorithm to obtain the first flow message. Fixing this first flow message, we let the prover pick two distinct random challenges and compute the corresponding responses. Then the prover commits to the response by querying $\mathcal{G}_{\mathsf{oRO}}$ and using the answer as the commitment. Next the prover sends the first flow message along with all the challenges and the commitments to the verifier. After that, the verifier asks the prover to open one commitment. Finally the verifier receives the response, and checks if the corresponding transcript is valid. The soundness error of the protocol described above is $\frac{1}{2}$, and it can be reduced by parallel repetitions. We also apply Fiat-Shamir transformation to remove the interaction [FS87]. The straight-line extractablity relies on the observability provided by $\mathcal{G}_{\mathsf{oRO}}$ and 2-special soundness.

Theorem 5 ([Pas03]). *Assume there is a 2-special sound SHVZK protocol, then there exists a straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world.*

If we instantiate the 2-special sound SHVZK protocol with ours that depicted in Figure 10, then the resulting straight-line extractable NIWH argument also only requires Minicrypt assumptions. We show a more general case that transforms a k-special sound SHVZK protocol for $k \geq 2$, into a straight-line NIWH argument in Appendix B.

5 Concluding Remarks: Towards a Complete Picture

In this work, we mainly focus on the lower bounds on round complexity for GUC-secure commitment protocols in the global random oracle models. We also wonder if such lower bounds exist, is it possible to construct round-optimal GUC-secure commitment protocols via Minicrypt assumptions?

In terms of the $\mathcal{G}_{\mathsf{oRO}}$, our work gives a complete answer: we show it is impossible to construct 2-round GUC-secure commitment in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world against static adversaries in Section 3, and construct a 3-round (round-optimal) GUC-secure commitment protocol via Minicrypt in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world in Section 4. In the remaining, let us turn our attention on other global random oracle models.

As for the $\mathcal{G}_{\mathsf{sRO}}$, the results of [CDPW07] rules out the possibility of constructing any GUC-secure commitment protocol in the $\mathcal{G}_{\mathsf{sRO}}$ hybrid world. More precisely, they argued that no "public setup", namely no setup that provides only public information that is available to all parties, can suffice for realizing commitment protocols in the GUC framework. It is easy to see that this impossibility result holds in the $\mathcal{G}_{\mathsf{sRO}}$ hybrid world.

Regarding the $\mathcal{G}_{\mathsf{poRO}}$, non-interactive GUC-secure commitment protocol can be achieved. In fact, Camenisch *et al.* proposed a non-interactive GUC-secure commitment in the $\mathcal{G}_{\mathsf{poRO}}$ hybrid world without any further assumptions [CDG+18].

Among all the global random oracle models depicted in Figure 4, only the $\mathcal{G}_{\mathsf{pRO}}$ has yet to be fully investigated. Actually, we already have some impossibility result: we find that there exists *no* GUC-secure commitment protocols with one-round committing phase in the $\mathcal{G}_{\mathsf{pRO}}$ hybrid world against static adversaries. Intuitively, we observe that the receiver does not have the chance to send any message in the committing phase in such commitment protocols. Note that, the $\mathcal{G}_{\mathsf{pRO}}$ only allows the simulator to program on the unqueried points without being detected, and the simulator benefits itself by letting the corrupted parties to work on its programmed points. Now let us consider the case where the committer is corrupted and the simulator acts as the receiver, the

simulator needs to extract the committed value before the opening phase. In a commitment protocol where the committing phase only takes one round, the simulator (acting as the receiver) does not need to send any message, thus it cannot enforce the corrupted committer to produce its message on the programmed points. If the simulator still succeeds in extracting the committed value from the commitment message, then we can use such a simulator to break the hiding property of the commitment scheme since anyone can run this simulator without relying on the programmability of the $\mathcal{G}_{\mathsf{pRO}}$. In conclusion, the committing phase requires at least 2 rounds, plus (at least) 1 round of the opening phase, and the entire commitment protocol requires at least 3 rounds. We refer interesting readers to see the formal theorem and proof in Appendix A.1.

Given this lower bound in the $\mathcal{G}_{\mathsf{pRO}}$, we find the 3-round (2 rounds for the committing phase, 1 round for the opening phase) GUC-secure commitment protocol proposed in [CDG+18] is round-optimal. But their construction relies on CDH assumption which lives in the Cryptomania world. The sad truth is that we find it extremely hard to construct a round-optimal GUC-secure commitment protocol via Minicrypt in the $\mathcal{G}_{\mathsf{pRO}}$ hybrid world, so we leave it as an open question.

References

[AHIV17] Scott Ames, Carmit Hazay, Yuval Ishai, and Muthuramakrishnan Venkitasubramaniam. Ligero: Lightweight sublinear arguments without a trusted setup. In *ACM CCS 2017*, pages 2087–2104. ACM Press, 2017.

[BBHR19] Eli Ben-Sasson, Iddo Bentov, Yinon Horesh, and Michael Riabzev. Scalable zero knowledge with no trusted setup. In *Crypto 2019, Part III*, volume 11694 of *LNCS*, pages 701–732. Springer, 2019.

[BCNP04] Boaz Barak, Ran Canetti, Jesper Buus Nielsen, and Rafael Pass. Universally composable protocols with relaxed set-up assumptions. In *FOCS 2004*, pages 186–195. IEEE Computer Society Press, 2004.

[BDD20] Carsten Baum, Bernardo David, and Rafael Dowsley. Insured MPC: Efficient secure computation with financial penalties. In *FC 2020*, volume 12059 of *LNCS*, pages 404–420. Springer, 2020.

[BGM19] Pedro Branco, Manuel Goulão, and Paulo Mateus. UC-commitment schemes with phase-adaptive security from trapdoor functions. Cryptology ePrint Archive, Report 2019/529, 2019. https://eprint.iacr.org/2019/529.

[BPRS17] Megha Byali, Arpita Patra, Divya Ravi, and Pratik Sarkar. Fast and universally-composable oblivious transfer and commitment scheme with adaptive security. Cryptology ePrint Archive, Report 2017/1165, 2017. https://eprint.iacr.org/2017/1165.

[BR93] Mihir Bellare and Phillip Rogaway. Random oracles are practical: A paradigm for designing efficient protocols. In *ACM CCS 1993*, pages 62–73. ACM Press, 1993.

[Bra21] Pedro Branco. A post-quantum UC-commitment scheme in the global random oracle model from code-based assumptions. *Advances in Mathematics of Communications*, 15(1):113, 2021.

[Can01] Ran Canetti. Universally composable security: A new paradigm for cryptographic protocols. In *FOCS 2001*, pages 136–145. IEEE Computer Society Press, 2001.

[CDG+17] Melissa Chase, David Derler, Steven Goldfeder, Claudio Orlandi, Sebastian Ramacher, Christian Rechberger, Daniel Slamanig, and Greg Zaverucha. Post-quantum zero-knowledge and signatures from symmetric-key primitives. In *ACM CCS 2017*, pages 1825–1842. ACM Press, 2017.

[CDG+18] Jan Camenisch, Manu Drijvers, Tommaso Gagliardoni, Anja Lehmann, and Gregory Neven. The wonderful world of global random oracles. In *Eurocrypt 2018, Part I*, volume 10820 of *LNCS*, pages 280–312. Springer, 2018.

[CDPW07] Ran Canetti, Yevgeniy Dodis, Rafael Pass, and Shabsi Walfish. Universally composable security with global setup. In *TCC 2007*, volume 4392 of *LNCS*, pages 61–85. Springer, 2007.

[CF01] Ran Canetti and Marc Fischlin. Universally composable commitments. In *Crypto 2001*, volume 2139 of *LNCS*, pages 19–40. Springer, 2001.

[CGH98] Ran Canetti, Oded Goldreich, and Shai Halevi. The random oracle methodology, revisited (preliminary version). In *ACM STOC 1998*, pages 209–218. ACM Press, 1998.

[CJS14] Ran Canetti, Abhishek Jain, and Alessandra Scafuro. Practical UC security with a global random oracle. In *ACM CCS 2014*, pages 597–608. ACM Press, 2014.

[COS20] Alessandro Chiesa, Dev Ojha, and Nicholas Spooner. Fractal: Post-quantum and transparent recursive proofs from holography. In *Eurocrypt 2020, Part I*, volume 12105 of *LNCS*, pages 769–793. Springer, 2020.

[CSW20] Ran Canetti, Pratik Sarkar, and Xiao Wang. Efficient and round-optimal oblivious transfer and commitment with adaptive security. In *Asiacrypt 2020, Part III*, volume 12493 of *LNCS*, pages 277–308. Springer, 2020.

[Dam02] Ivan Damgård. On σ-protocols. https://www.cs.au.dk/~ivan/Sigma.pdf.

[dOT21] Cyprien de Saint Guilhem, Emmanuela Orsini, and Titouan Tanguy. Limbo: Efficient zero-knowledge MPCitH-based arguments. In *ACM CCS 2021*, pages 3022–3036. ACM Press, 2021.

[DSW08] Yevgeniy Dodis, Victor Shoup, and Shabsi Walfish. Efficient constructions of composable commitments and zero-knowledge proofs. In *Crypto 2008*, volume 5157 of *LNCS*, pages 515–535. Springer, 2008.

[FS87] Amos Fiat and Adi Shamir. How to prove yourself: Practical solutions to identification and signature problems. In *Crypto 1986*, volume 263 of *LNCS*, pages 186–194. Springer, 1987.

[FS90] Uriel Feige and Adi Shamir. Witness indistinguishable and witness hiding protocols. In *ACM STOC 1990*, pages 416–426. ACM Press, 1990.

[GIS18] Sanjam Garg, Yuval Ishai, and Akshayaram Srinivasan. Two-round MPC: Information-theoretic and black-box. In *TCC 2018, Part I*, volume 11239 of *LNCS*, pages 123–151. Springer, 2018.

[GKPS18] Chaya Ganesh, Yashvanth Kondi, Arpita Patra, and Pratik Sarkar. Efficient adaptively secure zero-knowledge from garbled circuits. In *PKC 2018, Part II*, volume 10770 of *LNCS*, pages 499–529. Springer, 2018.

[GMO16] Irene Giacomelli, Jesper Madsen, and Claudio Orlandi. ZKBoo: Faster zero-knowledge for Boolean circuits. In *USENIX Security 2016*, pages 1069–1083. USENIX Association, 2016.

[GMR89] Shafi Goldwasser, Silvio Micali, and Charles Rackoff. The knowledge complexity of interactive proof systems. *SIAM Journal on Computing*, 18(1):186–208, 1989.

[GMW87] Oded Goldreich, Silvio Micali, and Avi Wigderson. How to play any mental game or A completeness theorem for protocols with honest majority. In *ACM STOC 1987*, pages 218–229. ACM Press, 1987.

[HM04] Dennis Hofheinz and Jörn Müller-Quade. Universally composable commitments using random oracles. In *TCC 2004*, volume 2951 of *LNCS*, pages 58–76. Springer, 2004.

[IKOS07] Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, and Amit Sahai. Zero-knowledge from secure multiparty computation. In *ACM STOC 2007*, pages 21–30. ACM Press, 2007.

[Imp95] Russell Impagliazzo. A personal view of average-case complexity. In *SCT 1995*, pages 134–147. IEEE, 1995.

[KKW18] Jonathan Katz, Vladimir Kolesnikov, and Xiao Wang. Improved non-interactive zero knowledge with applications to post-quantum signatures. In *ACM CCS 2018*, pages 525–537. ACM Press, 2018.

[KL20] Jonathan Katz and Yehuda Lindell. *Introduction to modern cryptography*. CRC press, 2020.

[KZ20] Benjamin Kuykendall and Mark Zhandry. Towards non-interactive witness hiding. In *TCC 2020, Part I*, volume 12550 of *LNCS*, pages 627–656. Springer, 2020.

[LR22] Anna Lysyanskaya and Leah Namisa Rosenbloom. Universally composable sigma-protocols in the global random-oracle model. Cryptology ePrint Archive, Report 2022/290, 290. https://eprint.iacr.org/2022/290.

[MR19] Daniel Masny and Peter Rindal. Endemic oblivious transfer. In *ACM CCS 2019*, pages 309–326. ACM Press, 2019.

[MRS17] Payman Mohassel, Mike Rosulek, and Alessandra Scafuro. Sublinear zero-knowledge arguments for RAM programs. In *Eurocrypt 2017, Part I*, volume 10210 of *LNCS*, pages 501–531. Springer, 2017.

[MY04] Philip D. MacKenzie and Ke Yang. On simulation-sound trapdoor commitments. In *Eurocrypt 2004*, volume 3027 of *LNCS*, pages 382–400. Springer, 2004.

[Pas03] Rafael Pass. On deniability in the common reference string and random oracle model. In *Crypto 2003*, volume 2729 of *LNCS*, pages 316–337. Springer, 2003.

[Yao82] Andrew Chi-Chih Yao. Protocols for secure computations (extended abstract). In *FOCS 1982*, pages 160–164. IEEE Computer Society Press, 1982.

A Lower Bounds on Round Complexity for GUC-Secure Commitment/ZK in the GPRO Model

A.1 Result for Commitment

Here we show that it is impossible to achieve GUC-secure commitment with one round for the committing phase in the $\mathcal{G}_{\mathsf{pRO}}$ hybrid world against static adversaries. We stress that this result is stronger than Theorem 1, since this result rules out the possibility of a commitment scheme which consists of one round for the committing phase and multiple rounds for the opening phase. We first provide a traditional commitment functionality $\mathcal{F}_{\mathsf{COM}}$ in Figure 12, since $\mathcal{F}_{\mathsf{tCOM}}$ does not fit the $\mathcal{G}_{\mathsf{pRO}}$ hybrid world.

Functionality $\mathcal{F}_{\mathsf{COM}}$

The functionality interacts with two parties C, R and an adversary $\mathcal{S}$.

- Upon receiving $(\textsc{Commit}, \mathsf{sid}, C, R, m)$ from C, do:
  - Record the tuple (sid, C, R, m), and send $(\textsc{Receipt}, \mathsf{sid}, C, R)$ to R and $\mathcal{S}$.
  - Ignore any subsequent $\textsc{Commit}$ command.
- Upon receiving $(\textsc{Decommit}, \mathsf{sid}, C, R)$ from C, do:
  - If there is a tuple (sid, C, R, m) recorded, send $(\textsc{Decommit}, \mathsf{sid}, C, R, m)$ to R and $\mathcal{S}$, and halt.
  - Otherwise, ignore the message.

Figure 12: The Functionality $\mathcal{F}_{\mathsf{COM}}$ for Commitment

Theorem 6. *There exists no terminating single-message (only restricted in the committing phase) protocol* Π *that GUC-realizes* $\mathcal{F}_{\mathsf{COM}}$ *depicted in Figure 12 with static security, using only the shared functionality for global programmable random oracle* $\mathcal{G}_{\mathsf{pRO}}$.

Proof. We proceed by contradiction. Suppose there exists such a protocol Π that GUC-realizes $\mathcal{F}_{\mathsf{COM}}$ in the $\mathcal{G}_{\mathsf{pRO}}$ hybrid world. Then there must exist a PPT simulator $\mathcal{S}$ such that $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{pRO}}}_{\mathcal{F}_{\mathsf{COM}},\mathcal{S},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{pRO}}}_{\Pi,\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{G}_{\mathsf{pRO}}$-externally constraint environment $\mathcal{Z}$.

In particular, let us consider the session with SID sid_1, and let $\mathcal{A}$ be a dummy adversary that simply forwards protocol flows between corrupt parties and the environment $\mathcal{Z}$. Let $\mathcal{Z}$ corrupt the committer C^* at first. Then $\mathcal{Z}$ chooses a random bit $b \in \{0,1\}$ and instructs C^* to execute the honest committer algorithm to produce the commitment ψ on input b, which we denote by $\psi \leftarrow C(\mathsf{sid}_1, b, \mathcal{Q}_{\mathsf{sid}_1,C})$, where $\mathcal{Q}_{\mathsf{sid}_1,C}$ is the queries to $\mathcal{G}_{\mathsf{pRO}}$ sent by C^*. In the opening phase, $\mathcal{Z}$ instructs C^* to reveal the committed value b and waits for R to output b'. If $b = b'$, $\mathcal{Z}$ outputs 1; otherwise, $\mathcal{Z}$ outputs 0.

In order to make the GUC experiments above remain indistinguishable, the simulator $\mathcal{S}$ needs to extract the committed bit b from the commitment ψ sent by C^*, and send $(\mathsf{Commit}, \mathsf{sid}_1, C, R, b)$ to $\mathcal{F}_{\mathsf{COM}}$ on behalf of the dummy committer. Recall that, the main advantage of $\mathcal{S}$ is that it can program the $\mathcal{G}_{\mathsf{pRO}}$ on unqueried points without being detected, and we denote by $\mathsf{Prog}_{\mathsf{sid}_1}$ the queries programmed by $\mathcal{S}$. If the adversary happens to use the points that belongs to $\mathsf{Prog}_{\mathsf{sid}_1}$, then $\mathcal{S}$ has the chance of extracting the private information of the adversary. Note that, the simulator $\mathcal{S}$ also can query $\mathcal{G}_{\mathsf{pRO}}$ just like normal parties. In order to describe the process of querying to $\mathcal{G}_{\mathsf{pRO}}$, we denote by $\mathcal{G}^*_{\mathsf{pRO}}$ the simplified version of the $\mathcal{G}_{\mathsf{pRO}}$, that is, the $\mathcal{G}_{\mathsf{pRO}}$ without the $\textsc{Program}$ interface. We write $\mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}$ to denote the event that $\mathcal{S}$ is given query oracle access to $\mathcal{G}_{\mathsf{pRO}}$ and

can continuously query to $\mathcal{G}_{\mathsf{pRO}}$. With notations above, we will write $b \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_1, \psi, \mathsf{Prog}_{\mathsf{sid}_1})$ to denote the event where $\mathcal{S}$ extracts the committed bit b from ψ using $\mathsf{Prog}_{\mathsf{sid}_1}$ after querying to $\mathcal{G}_{\mathsf{pRO}}$. We note that, $\mathcal{S}$ should be able to handle any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{Z}$. Consider such a case where $\mathcal{Z}$ performs the following attack: $\mathcal{Z}$ queries $\mathcal{G}_{\mathsf{pRO}}$ everything that will be needed in advance (recall that, these queries are denoted as $\mathcal{Q}_{\mathsf{sid}_1,C}$), then starts the protocol Π and instructs C^* to run the honest committer algorithm on those previously queried points. Since we are in a commitment protocol where committing phase only takes one round, this attack can be performed successfully. In such a case, we have $\Pr[\mathsf{Prog}_{\mathsf{sid}_1} \cap \mathcal{Q}_{\mathsf{sid}_1,C} \neq \emptyset] = 0$, where $\mathcal{Q}_{\mathsf{sid}_1,C}$ is the queries used for generating ψ. In other words, the simulator $\mathcal{S}$ has no advantages at all in this case. For notation convenience, we denote by $b \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_1, \psi, \emptyset)$ the event where $\mathcal{S}$ is still able to extract b from ψ even if $\mathsf{Prog}_{\mathsf{sid}_1} \cap \mathcal{Q}_{\mathsf{sid}_1,C} = \emptyset$. We note that, the algorithm $\mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_1, \psi, \emptyset)$ does not use the PROGRAM interface at all and can be run by anyone, since anyone can query to $\mathcal{G}_{\mathsf{pRO}}$. We also note that, even if we switch to a session with a different SID, the algorithm $\mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_1, \psi, \emptyset)$ still works as long as the appropriate inputs are provided.

In the following, we show that the existence of the simulator $\mathcal{S}$ above contradicts the security of Π against static corruptions, by creating a particular environment $\mathcal{Z}'$ which succeeds in distinguishing $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{pRO}}}_{\mathcal{F}_{\mathsf{COM}},\mathcal{S}',\mathcal{Z}'}$ from $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{pRO}}}_{\Pi,\mathcal{A},\mathcal{Z}'}$ after a static corruption operation for any PPT simulator $\mathcal{S}'$. Now consider the session with SID sid_2. We let $\mathcal{Z}'$ corrupt the receiver R^* at first, and feed the honest committer C with a randomly selected bit $b \in \{0,1\}$, finally wait for R^* to output $(\text{RECEIPT}, \mathsf{sid}_2, C, R)$. In this case, the simulator $\mathcal{S}'$ needs to produces an equivocal commitment ψ without knowing b. In other words, the entire computation of ψ is totally independent of b. After receiving ψ from the committer, $\mathcal{Z}'$ simply runs $b' \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}, \psi, \emptyset)$. In the real-world, we always have $b' = b$. In the ideal-world, since the entire computation of ψ is independently of b, we have $b' = b$ with probability at most $\frac{1}{2}$. Therefore, $\mathcal{Z}'$ can distinguish between the real-world and the ideal-world experiments at least $\frac{1}{2}$, contradicting our assumption that Π is GUC-secure. □

A.2 Result for ZK

We show it is impossible to achieve GUC-secure NIZK protocols for *non-trivial* NP relations in the $\mathcal{G}_{\mathsf{pRO}}$ hybrid world against static adversaries. We first provide the ZK functionality $\mathcal{F}_{\mathsf{ZK}}$ in Figure 13.

Functionality $\mathcal{F}_{\mathsf{ZK}}$

The functionality interacts with two parties P, V and an adversary $\mathcal{S}$. It is parameterized by an NP relation $\mathcal{R}_{\mathcal{L}}$.

- Upon receiving (ZK-PROVER, sid, P, V, x, w) from P, do:
  - Assert $\mathcal{R}_{\mathcal{L}}(x, w) = 1$.
  - Output (ZK-PROOF, sid, P, V, x) to V and the adversary $\mathcal{S}$.

Figure 13: The Functionality $\mathcal{F}_{\mathsf{ZK}}$ for Zero-Knowledge

Definition 12. *We say an NP relation $\mathcal{R}_{\mathcal{L}}$ whose associate language is $\mathcal{L}$ is non-trivial if there is no PPT algorithm efficiently decides membership in $\mathcal{L}$ (i.e. $\mathcal{L} \notin BPP$). Furthermore, we say $\mathcal{R}_{\mathcal{L}}$ is non-trivial with respect to shared functionality $\mathcal{G}$ if there is no PPT algorithm efficiently decides membership in $\mathcal{L}$ even when given oracle access to $\mathcal{G}$.*

Theorem 7. *There exists no terminating one-round protocol* Π *that GUC-realizes* $\mathcal{F}_{\mathsf{ZK}}$ *depicted in Figure 13 with static security, using only the shared functionality for global programmable random oracle* $\mathcal{G}_{\mathsf{pRO}}$*, for any NP relation* $\mathcal{R}_{\mathcal{L}}$ *that is non-trivial with respect to* $\mathcal{G}_{\mathsf{pRO}}$*.*

Proof. We proceed by contradiction. Suppose there exists such a protocol Π that GUC-realizes $\mathcal{F}_{\mathsf{ZK}}$ in the $\mathcal{G}_{\mathsf{pRO}}$ hybrid world for an NP relation $\mathcal{R}_{\mathcal{L}}$ whose s language is $\mathcal{L}$. Then there must exist a PPT simulator $\mathcal{S}$ such that $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{pRO}}}_{\mathcal{F}_{\mathsf{ZK}},\mathcal{S},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{pRO}}}_{\Pi,\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{G}_{\mathsf{pRO}}$-externally constraint environment $\mathcal{Z}$.

Let us consider the session with SID sid_1, and let $\mathcal{A}$ be a dummy adversary that simply forwards protocol flows between corrupt parties and the environment $\mathcal{Z}$. Let $\mathcal{Z}$ corrupt the prover P^* at first. Then $\mathcal{Z}$ chooses $(x, w) \in \mathcal{R}_{\mathcal{L}}$ as the input, instructs P^* to run the honest prover algorithm $\pi \leftarrow P(\mathsf{sid}_1, x, w, \mathcal{Q}_{\mathsf{sid}_1,P})$, where $\mathcal{Q}_{\mathsf{sid}_1,P}$ is the queries sent by P^*. Then $\mathcal{Z}$ waits for V to output x'. If $x = x'$, $\mathcal{Z}$ outputs 1; otherwise, $\mathcal{Z}$ outputs 0.

In order to make the GUC experiments above remain indistinguishable, the simulator $\mathcal{S}$ needs to extract a valid witness w from the proof π sent by P^* such that $(x, w) \in \mathcal{R}_{\mathcal{L}}$, and then sends $(\text{ZK-PROVER}, \mathsf{sid}, P, V, x, w)$ to $\mathcal{F}_{\mathsf{ZK}}$ on behalf of the dummy prover; otherwise, $\mathcal{Z}'$ can successfully distinguish the two experiments, contradicting the GUC-security of the protocol Π. Recall that, the main advantage of $\mathcal{S}$ is that it can program the $\mathcal{G}_{\mathsf{pRO}}$ on unqueried points without being detected, and we denote by $\mathsf{Prog}_{\mathsf{sid}_1}$ the queries programmed by $\mathcal{S}$. If the adversary happens to use the points that belongs to $\mathsf{Prog}_{\mathsf{sid}_1}$, then $\mathcal{S}$ has the chance of extracting the private information of the adversary. Note that, the simulator $\mathcal{S}$ also can query $\mathcal{G}_{\mathsf{pRO}}$ just like normal parties. In order to describe the process of querying to $\mathcal{G}_{\mathsf{pRO}}$, we denote by $\mathcal{G}^*_{\mathsf{pRO}}$ the simplified version of the $\mathcal{G}_{\mathsf{pRO}}$, that is, the $\mathcal{G}_{\mathsf{pRO}}$ without the PROGRAM interface. We write $\mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}$ to denote the event that $\mathcal{S}$ is given query oracle access to $\mathcal{G}_{\mathsf{pRO}}$ and can continuously query to $\mathcal{G}_{\mathsf{pRO}}$. With notions above, we can write $w \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_1, x, \pi, \mathsf{Prog}_{\mathsf{sid}_1})$ to denote the event where $\mathcal{S}$ extracts the witness w from the proof π using $\mathsf{Prog}_{\mathsf{sid}_1}$ after querying to $\mathcal{G}_{\mathsf{pRO}}$. Analogous to the discussion of Theorem 6, $\mathcal{S}$ should handle the case where $\Pr[\mathsf{Prog}_{\mathsf{sid}_1} \cap \mathcal{Q}_{\mathsf{sid}_1,P} \neq \emptyset] = 0$ where $\mathcal{Q}_{\mathsf{sid}_1,P}$ is the queries used for generating the proof π, and we write $b \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_1, x, \pi, \emptyset)$ to denote the event where $\mathcal{S}$ is still able to extract b from π even if $\mathsf{Prog}_{\mathsf{sid}_1} \cap \mathcal{Q}_{\mathsf{sid}_1,P} = \emptyset$. We note that, the algorithm $\mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_1, x, \pi, \emptyset)$ can be run by anyone, since the algorithm does not use the PROGRAM interface at all and anyone can query to $\mathcal{G}_{\mathsf{pRO}}$. We also note that, even if we switch to a session with a different SID, the algorithm $\mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_1, x, \pi, \emptyset)$ still works as long as the appropriate inputs are provided.

In the following, we consider another PPT $\mathcal{G}_{\mathsf{oRO}}$-externally constrained environment $\mathcal{Z}'$ and PPT simulator $\mathcal{S}'$. Now consider a session with SID sid_2. We let $\mathcal{Z}'$ corrupt the verifier V^* at first, and feed the honest prover P with $(x, w) \in \mathcal{R}_{\mathcal{L}}$. Then $\mathcal{Z}$ waits for P to send the proof π. If π is valid, $\mathcal{Z}$ outputs 1; otherwise, $\mathcal{Z}$ outputs 0.

In this case, $\mathcal{S}'$ needs to simulate an accepting proof π without w, since w is the hidden input to the honest party. Similarly, we write $\pi \leftarrow \mathcal{S}'^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_2, x, \mathsf{Prog}_{\mathsf{sid}_2})$ to denote the event where the simulator $\mathcal{S}'$ can produce the proof π without the witness w with the aid of the programmed queries $\mathsf{Prog}_{\mathsf{sid}_2}$. We note that, the entire computation of π is totally independent of w since $\mathcal{F}_{\mathsf{ZK}}$ hides w from $\mathcal{S}'$ information theoretically. We also note that $\mathcal{S}'^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}_2, x, \mathsf{Prog}_{\mathsf{sid}_2})$ still works as long as the appropriate inputs are provided, even if we switch to the session with another SID.

To conclude the proof, we show there exists a PPT decider $\mathcal{D}$ that can efficiently compute the witness w for any given statement $x \in \mathcal{L}$ using only $\mathcal{G}_{\mathsf{pRO}}$. Given any statement x, $\mathcal{D}$ first select a party to act as P simulated by $\mathcal{S}'$ and a party to act as V^* controlled by $\mathcal{A}/\mathcal{Z}$, and starts the protocol Π in the session with SID sid. First of all, we let $\mathcal{S}'$ program $\mathcal{G}_{\mathsf{pRO}}$ on queries $\mathsf{Prog}_{\mathsf{sid}}$, and let $\mathcal{S}'$ run $\mathcal{S}'^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}, x, \mathsf{Prog}_{\mathsf{sid}})$ to produce the simulated proof π. Then we let V^* run $\mathcal{S}^{\mathcal{G}^*_{\mathsf{pRO}}}(\mathsf{sid}, x, \pi, \emptyset)$

to output the witness w. Finally, we examine if the extracted w is valid: if $(x, w) \in \mathcal{R}_{\mathcal{L}}$, $\mathcal{D}$ outputs 1 indicating $x \in \mathcal{L}$; otherwise, $\mathcal{D}$ outputs 0 indicating $x \notin \mathcal{L}$. It is easy to see that when $x \in \mathcal{L}$, $\mathcal{D}$ always outputs 1; when $x \notin \mathcal{L}$, $\mathcal{D}$ outputs 1 with only negligible probability. Therefore, we have successfully constructed a PPT decider $\mathcal{D}$ that efficiently decides membership in $\mathcal{L}$ (i.e. $\mathcal{L} \in BPP$) when given oracle access to $\mathcal{G}_{\mathsf{pRO}}$. This contradicts the non-triviality of $\mathcal{R}_{\mathcal{L}}$ w.r.t. $\mathcal{G}_{\mathsf{pRO}}$. □

B Straight-line Extractable NIWH Argument from k-Special Sound SHVZK Protocol

In [Pas03], Pass showed how to transform a 2-special sound SHVZK protocol into a straight-line extractable NIWH argument in the observable RO model. Here we generalize the idea of Pass and construct the straight-line extractable NIWH argument from k-special sound SHVZK protocol where $k \geq 2$.

Let P be the prover algorithm and V be the verifier algorithm. Let $\ell = \min\{\log_k(2^{\ell_{\mathsf{out}}(\lambda)}), \log_k(2^{\lambda})\}$ be the repetition parameter. We denote the event where queries $\mathcal{G}_{\mathsf{oRO}}$ on input x and gets the answer y as $y := \mathsf{oRO}(x)$. Formally, we present our protocol Π_{NIWH} in Figure 14 and prove the security through Theorem 8. If we use our SHVZK protocol Π_{SHVZK} depicted in Figure 10 as the main building block, then we can obtain a NIWH argument via only Minicrypt assumptions in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world.

Protocol Π_{NIWH}

Primitives: SHVZK protocol $\Pi_{\mathsf{SHVZK}} = \Pi_{\mathsf{SHVZK}}.\{\mathsf{Move1}, \mathsf{Move2}, \mathsf{Move3}, \mathsf{Verify}, \mathsf{Sim}\}$ and one-way function g.
Inputs: P and V has a common y. P has a private seed such that $y = g(\mathsf{seed})$.

Proof Generation: $\mathsf{Prove}^{\mathcal{G}_{\mathsf{oRO}}}(y, \mathsf{seed})$:

- For $i \in [\ell]$:
  - Select λ-bit random $r_i, \{s_{i,j}\}_{j\in[k]}$, and compute $a_i := \Pi_{\mathsf{SHVZK}}.\mathsf{Move1}(y, \mathsf{seed}; r_i)$ for relation $\mathcal{R}_1 = \{(y, \mathsf{seed}) \mid y = g(\mathsf{seed})\}$.
  - Select k distinct λ-bit random challenge $\{e_{i,j}\}_{j\in[k]}$.
  - For $j \in [k]$: compute the corresponding response $z_{i,j} := \Pi_{\mathsf{SHVZK}}.\mathsf{Move3}(y, \mathsf{seed}, e_{i,j}; r_i)$ for relation $\mathcal{R}_1$, and commit to the response $\mathsf{com}_{i,j} := \mathsf{oRO}(\mathsf{sid}, \text{`}P\text{'}||z_{i,j}||s_{i,j})$.
- Set $a = (\{a_i, \{e_{i,j}, \mathsf{com}_{i,j}\}_{j\in[k]}\}_{i\in[\ell]})$, and compute $h := \mathsf{oRO}(\mathsf{sid}, \text{`}P\text{'}||y||a)$.
- Convert h into a k-ary number, and use h_i to represent the i-th digit of this k-ary number.
- For $i \in [\ell]$: set $z_i := z_{i,h_i+1}$, $s_i := s_{i,h_i+1}$.
- Set $z := \{z_i, s_i\}_{i\in[\ell]}$ and send $\pi := (a, z)$.

Verification: $\mathsf{Verify}^{\mathcal{G}_{\mathsf{oRO}}}(y, \pi)$:

- Compute $h := \mathsf{oRO}(\mathsf{sid}, \text{`}P\text{'}||y||a)$, convert h into a k-ary number, and use h_i to represent the i-th digit of this k-ary number.
- For $i \in [\ell]$:
  - Check if $\{e_{i,j}\}_{j\in[k]}$ are distinct.
  - Check if the following conditions hold: $\mathsf{com}_{i,h_i+1} = \mathsf{oRO}(\mathsf{sid}, \text{`}P\text{'}||z_i||s_i)$ and $\Pi_{\mathsf{SHVZK}}.\mathsf{Verify}(y, a_i, e_{i,h_i+1}, z_i) = 1$ for relation $\mathcal{R}_1$.
- If all the checks pass, output 1; otherwise, output 0.

Figure 14: Protocol Π_{NIWH} in the $\mathcal{G}_{\mathsf{oRO}}$ Hybrid World for Proving $y = g(\mathsf{seed})$

Theorem 8. *Assume g is a one-way function that is hard to invert. Assume Π_{SHVZK} is a k-special sound SHVZK protocol. The protocol Π_{NIWH} described in Figure 14 is a NIWH argument in the $\mathcal{G}_{\mathsf{oRO}}$ hybrid world which satisfies perfect completeness, computational soundness, witness hiding and straight-line extraction.*

Proof. Perfect Completeness. It is straightforward.

Witness Hiding. We first prove our protocol is witness hiding. We proceed by contradiction. Assume there exists a PPT adversary $\mathcal{A}$ that breaks the witness hiding property of Π_{NIWH} with non-negligible probability, then we are able to build a reduction $\mathcal{B}$ which violates the Hard to Invert (HI) property of the underlying g. First $\mathcal{B}$ interacts with the HI game challenger $\mathcal{C}$, and receives y from $\mathcal{C}$. Then $\mathcal{B}$ simulates $\mathcal{G}_{\mathsf{oRO}}$ and starts the protocol Π_{NIWH} with $\mathcal{A}$ by running $\mathcal{A}$ internally as black-box. Thus, our $\mathcal{B}$ sees all queries $\mathcal{A}$ makes to $\mathcal{G}_{\mathsf{oRO}}$ and produces their answers. The internal description of $\mathcal{B}$ follows:

- Select a random $h \leftarrow \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$, convert h into a k-ary number, and use h_i to represent the i-th digit of this k-ary number.
- For $i \in [\ell]$: select random $r_i, e_{i,h_i+1} \leftarrow \{0,1\}^\lambda$, and run $(a_i, z_{i,h_i+1}) := \Pi_{\mathsf{SHVZK}}.\mathsf{Sim}(y, e_{i,h_i+1}; r_i)$ for relation $\{(y, \mathsf{seed}) \mid y = g(\mathsf{seed})\}$. Select random $\{e_{i,j}, z_{i,j}\}_{j\in[k], j\neq h_i+1} \leftarrow \{0,1\}^\lambda$ such that $\{e_{i,j}\}_{j\in[k]}$ are distinct.
- For $i \in [\ell], j \in [k]$: select random $s_{i,j} \leftarrow \{0,1\}^\lambda$, and compute $\mathsf{com}_{i,j} := \mathsf{oRO}(\mathsf{sid}, `P'||z_{i,j}||s_{i,j})$.
- Set $a := (\{a_i, \{e_{i,j}, \mathsf{com}_{i,j}\}_{j\in[k]}\}_{i\in[\ell]})$, and program the answer of $\mathcal{G}_{\mathsf{oRO}}$ as h on query $(\mathsf{sid}, `P'||y||a)$. Set $z := (\{z_{i,h_i+1}, s_{i,h_i+1}\}_{i\in[\ell]})$.
- Send $(\mathsf{sid}, y, \pi := (a, z))$ to $\mathcal{A}$, and wait for $\mathcal{A}$ to output seed^*.

When $\mathcal{A}$ outputs seed^*, $\mathcal{B}$ checks if $y = g(\mathsf{seed}^*)$ holds. If so, $\mathcal{B}$ sends seed^* to the HI game challenger $\mathcal{C}$ and wins the game. Therefore, we prove that our protocol is witness-hiding.

Straight-line Extractability. We first show the strategy of the straight-line extractor $\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}$ which is granted the observability of $\mathcal{G}_{\mathsf{oRO}}$. Given a statement y and a potentially maliciously generated and *accepting* proof π, it works as follows:

- Receive the query-answer list of $\mathcal{G}_{\mathsf{oRO}}$.
- For $i \in [\ell]$: check if there exists a query of the form $(\mathsf{sid}, `P'||z_{i,j}||s_{i,j})$ such that $\mathsf{com}_{i,j} = \mathsf{oRO}(\mathsf{sid}, `P'||z_{i,j}||s_{i,j})$ for $j \in [k]$. If not, start over with the next i. Otherwise, extract seed_i from $\{(a_i, e_{i,j}, z_{i,j})\}_{j\in[k]}$ by k-special soundness of Π_{SHVZK}, and output $\mathsf{seed} := \mathsf{seed}_i$.
- Abort if all the steps above fails.

We then show the knowledge error of the straight-line extractor$\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}$ is negligible. In order to show that, we only have to prove that the straight-line extractor $\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}$ aborts at a negligible probability. Note that the straight-line extractor aborts when there exists a PPT adversary $\mathcal{A}$ which guesses h_i correctly, and runs $(a_i, z_{i,h_i+1}) := \Pi_{\mathsf{SHVZK}}.\mathsf{Sim}(y, e_{i,h_i+1}; r_i)$ to simulate the accepting proof. Due to the unpredictability of the output of $\mathcal{G}_{\mathsf{oRO}}$, this case happens at probability $k^{-\ell} = \max\{2^{-\ell_{\mathsf{out}}(\lambda)}, 2^{-\lambda}\}$ which is negligible. In other words, our $\Pi_{\mathsf{NIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{oRO}}}$ can output an extracted witness seed with overwhelming probability. Guaranteed by the k-special soundness of Π_{SHVZK}, the extracted seed must satisfy the condition $y = g(\mathsf{seed})$. Therefore, we prove that our protocol has a straight-line extractor with negligible knowledge error.

Computational Soundness. It is implied by straight-line extraction. □